VANDERBILT UNIVERSITY, MATH 2300-04, F 20
EXAMPLES OF SECTION 14.4

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Question 1. Given that f is a differentiable function with $f(2,5) = 6$, $f_x(2,5) = 1$, and $f_y(2,5) = -1$, estimate $f(2.2, 4.9)$.

Solution 1. Notice that the function f itself is not given, but we can estimate $f(2.2, 4.9)$ using a linear approximation since $(2.2, 4.9)$ is near $(2,5)$, and the value of f at $(2,5)$ is given.

We have the linear approximation

$$\begin{aligned} f(x,y) &\approx f(2,5) + f_x(2,5)(x-2) + f_y(2,5)(y-5) \\ &= 6 + 1(x-2) + (-1)(y-5) \\ &= x - y + 9. \end{aligned}$$

Thus

$$\begin{aligned} f(2.2, 4.9) &\approx 2.2 - 4.9 + 9 \\ &= 6.3. \end{aligned}$$