VANDERBILT UNIVERSITY, MATH 2300-04, F 20
EXAMPLES OF SECTION 14.2

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Question 1. Find the limit

$$\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$$

if it exists, or show that the limit does not exist.

Solution 1. Write $f(x,y) = \frac{xy^4}{x^2+y^8}$, so $f(x,0) = 0$ for $x \neq 0$ and thus $f(x,0) \to 0$ as $(x,y) \to (0,0)$ along the x-axis. Next, consider approaching $(0,0)$ along the curve $x = y^4$, so that $f(x,y) = f(y^4,y) = y^8/2y^8 = \frac{1}{2}$ for $y \neq 0$, thus $f(x,y) \to \frac{1}{2}$ along the curve $x = y^4$. Therefore the limit does not exits.

Question 2. Find the limit

$$\lim_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2}-1}{x^2+y^2}.$$

Solution 2. Set $r = \sqrt{x^2+y^2}$, so $r \to 0^+$ when $(x,y) \to (0,0)$. Then

$$\begin{aligned}
\lim_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2}-1}{x^2+y^2} &= \lim_{r\to 0^+} \frac{e^{-r^2}-1}{r^2} \\
&\to \frac{0}{0} \\
&\underbrace{=}_{\text{L'Hospital rule}} \lim_{r\to 0^+} \frac{e^{-r^2}(-2r)}{2r} \\
&= 0.
\end{aligned}$$