VANDERBILT UNIVERSITY, MATH 2300-04, F 20
EXAMPLES OF SECTION 13.3

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Question 1. Suppose you start at the point $(0,0,3)$ and move 5 units along the curve $x = 3\sin t$, $y = 4t$, $z = 3\cos t$ in the positive direction. Where are you now?

Solution 1. Write $\mathbf{r}(t) = \langle 3\sin t, 4t, 3\cos t\rangle$, so that $\mathbf{r}'(t) = \langle 3\cos t, 4, -3\sin t\rangle$ and $|\mathbf{r}'(t)| = 5$. The point $(0,0,3)$ corresponds to $t = 0$. Calculating the arc length:

$$s(t) = \int_0^t |\mathbf{r}'(\tau)|\, d\tau = 5t.$$

Since we moved five units along the curve, $s(t) = 5$, or $5t = 5 \Rightarrow t = 1$. Plugging $t = 1$ into $\mathbf{r}(t)$ gives $(3\sin 1, 4, 3\cos 1)$.

Question 2. Find the curvature of the ellipse $x = 3\cos t$, $y = 4\sin t$, $z = 0$ at the points $(3,0,0)$ and $(0,4,0)$.

Solution 2. Compute $\mathbf{r}'(t) = \langle -3\sin t, 4\cos t, 0\rangle$ and $\mathbf{r}''(t) = \langle -3\cos t, -4\sin t, 0\rangle$. Then

$$|\mathbf{r}'|^3 = (\sqrt{9\sin^2 t + 4\cos^2 t})^3,$$

and

$$\kappa(t) = \frac{|(-3\sin t)(-4\sin t) - 4\cos t(-3\cos t)|}{(\sqrt{9\sin^2 t + 4\cos^2 t})^3} = \frac{12}{(9\sin^2 t + 4\cos^2 t)^{\frac{3}{2}}}.$$

$(3,0,0)$ corresponds to $t = 0$, so $\kappa(0) = 3/16$; $(0,4,0)$ corresponds to $t = \pi/2$ so $\kappa(\pi/2) = 4/9$.