VANDERBILT UNIVERSITY, MATH 2300-04, F 20 EXAMPLES OF SECTION 12.6

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Question 1. Sketch the quadric

$$2x^2 + 4x - y^2 + 6y - z + 4z = 5.$$

Solution 1. Completing the square, we see that the equation can be written as

$$z - 2 = 2(x+1)^2 - (y-3)^2$$
.

Setting u = x + 1, v = y - 3, and w = z - 2, we have

$$w = 2u^2 - v^2,$$

Which we recognize as a hyperbolic paraboloid in the variables (u, v, w). Since (u, v, w) is obtained from (x, y, z) by a translation (the origin in the (u, v, w) system corresponds to the point (-1, 3, 2) in the original (x, y, z) variables), the original equation is also a hyperbolic paraboloid.

A few computer generated plots, from different angles, are given below.

