VANDERBILT UNIVERSITY, MATH 2300-04, F 20
EXAMPLES OF SECTION 12.2

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Question 1. Find $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-2\mathbf{b}$ if $\mathbf{a}=\langle 1,0,-1\rangle$ and $\mathbf{b}=\langle 3,-2,1\rangle$.

Solution 1. It is immediate to compute

$$\mathbf{a}+\mathbf{b}=\langle 1,0,-1\rangle+\langle 3,-2,1\rangle=\langle 4,-2,0\rangle,$$

and

$$\mathbf{a}-2\mathbf{b}=\langle 1,0,-1\rangle+2\langle 3,-2,1\rangle=\langle 7,-4,1\rangle.$$

Question 2. Find a vector in $\mathbb{R}^2$ that has length one and is parallel to the tangent line to the curve $y=2\sin x$ at $(\frac{\pi}{6},1)$. Is your answer unique?

Solution 2. The slope of the tangent line is

$$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}}=2\cos x|_{x=\frac{\pi}{6}}=\sqrt{3}.$$

Therefore, the vector $\mathbf{v}=\langle 1,\sqrt{3}\rangle$ is parallel to the tangent line to the curve at the given point. From Pythagoras, we see that the length of $\mathbf{v}$ is $\sqrt{1^2+\sqrt{3}^2}=2$. Hence, the vector

$$\mathbf{u}=\frac{1}{2}\mathbf{v}$$

has length one.

The answer is not unique since the vector $-\mathbf{u}$ shares the same properties.