VANDERBILT UNIVERSITY

MATH 2300 – MULTIVARIABLE CALCULUS

Practice Test 2

Directions. This practice test should be used as a study guide, illustrating the concepts that will be emphasized in the test. This does not mean that the actual test will be restricted to the content of the practice. Try to identify, from the questions below, the concepts and sections that you should master for the test. For each question in the practice test, study the ideas and techniques connected to the problem, even if they are not directly used in your solution.

Take this also as an opportunity to practice how you will write your solutions in the test. For this, write clearly, legibly, and in a logical fashion. Make precise statements (for instance, write an equal sign if two expressions are equal; say that one expression is a consequence of another when this is the case, etc).

Question 1. Express $\iiint_D f(x,y,z)\,dV$ as an iterated integral in six different ways, where D is the solid bounded by the given surfaces.

(a) $y = x^2$, $z = 0$, $y + 2z = 4$.

(b) $x = 2$, $y = 2$, $z = 0$, $x + y - 2z = 2$.

Question 2. Rewrite the integral

$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x, y, z)\, dy\, dz\, dx$$

as an equivalent iterated integral in the five other orders.

Question 3. (a) Write $\iiint\limits_D f(x,y,z)\,dV$ as an iterated integral in Cartesian coordinates in two different ways, where D is the region bounded by $z+x^2+y^2=1$, $y+x=0$, and the xy-plane.

(b) Using Cartesian, cylindrical, or spherical coordinates, evaluate the integral in (a) if $f(x,y,z) = \sqrt{x^2+y^2}$.

Question 4. A region R in the xy-plane is given. Find equations for a transformation T that maps a rectangular region S in the uv-plane onto R, where the sides of S are parallel to the u- and v-axes.

(a) R is bounded by $y = -x - 3$, $y = -5$, $y + x + 1 = 0$, and the x-axis.

(b) R is bounded by $y = 1/x$, $y = 4/x$, and the lines $y = x$ and $y = 4x$ in the first quadrant.

(c) R is bounded $y = \sin x + 2$, $x = 0$, $x = 2\pi$, and $y = \sin x$.

(d) R is bounded $x^2 + y^2 = 9$, $x^2 + y^2 = 1$, $y = x$, and $y = -x$ (this actually determines more than one region; choose one).

Question 5. Evaluate the integrals:

(a) $$\iint\limits_D \frac{\sqrt{x^2+16y^2+8xy}}{2\sqrt{x^2+4y^2}}\,dA,$$

where D is the region bounded by $x^2+4y^2=4$, $x^2+4y^2=16$, $y-x-1=0$, and $y-x+2=0$, and $x\geq 0$.

(b) $$\iint\limits_D (1-2y)\,dA,$$

where D is the region bounded by the curve $x+y^2=4$ and the line joining the points $(-5,-3)$ and $(0,2)$.

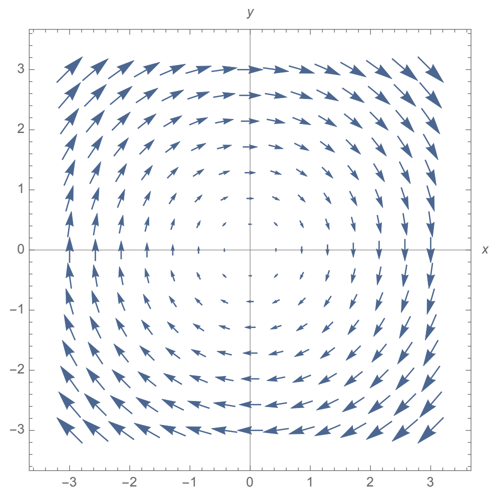
Question 6. Match the vector fields $\mathbf{F}$ with the given plots.

(a) $\mathbf{F}(x, y) = \cos x\,\mathbf{i} - \mathbf{j}$.

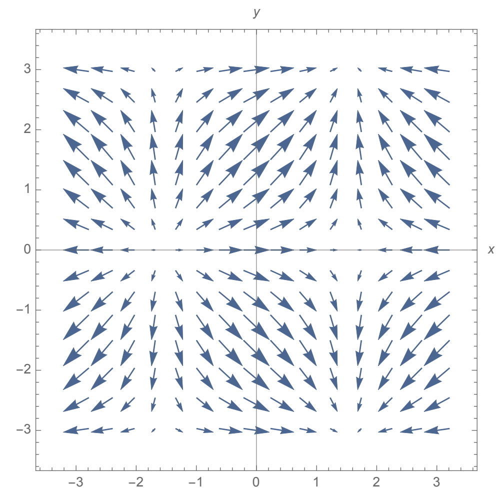
(b) $\mathbf{F}(x, y) = \frac{x}{2}\,\mathbf{i} + y\,\mathbf{j}$.

(c) $\mathbf{F}(x, y) = y\,\mathbf{i} - x\,\mathbf{j}$.

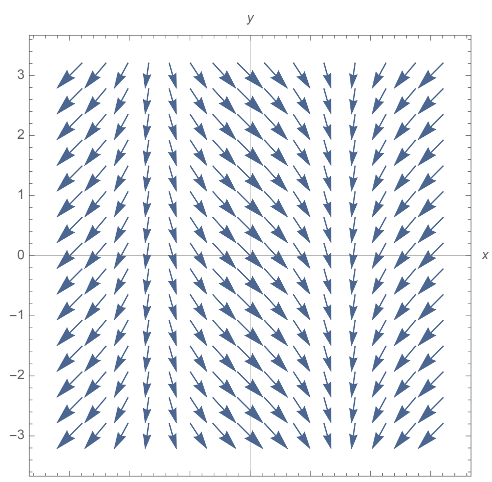
(d) $\mathbf{F}(x, y) = \cos x\,\mathbf{i} + \sin y\,\mathbf{j}$.



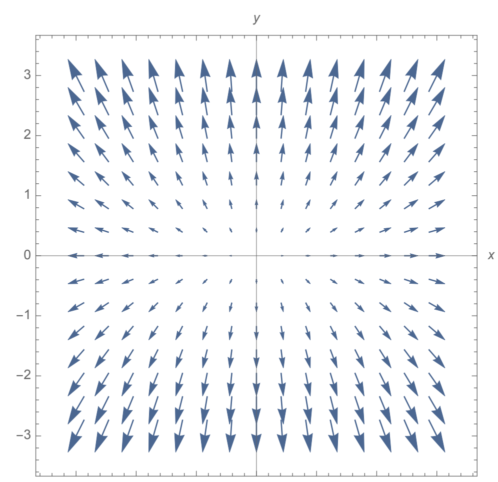
(I)



(II)



(III)



(IV)

Question 7. Determine whether or not the vector field $\mathbf{F}$ is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$.

(a) $\mathbf{F}(x, y) = (y^2 - 2x)\,\mathbf{i} + 2xy\,\mathbf{j}$.

(b) $\mathbf{F}(x, y) = \cos y\,\mathbf{i} + x \sin y\,\mathbf{j}$.

Question 8. Evaluate the line integral, where C is the given curve.

(a) $\displaystyle\int_C \frac{x}{y}\,ds$, where C is given by $x = t^3$, $y = t^4$, $1 \le t \le 2$.

(b) $\displaystyle\int_C (xy\,\mathbf{i} + \frac{x^2}{2}\,\mathbf{j}) \cdot d\mathbf{r}$.

The curve C is given by $C = C_1 \cup C_2 \cup C_3$, where C_1 is the line segment joining $(-2,-1)$ to $(-2,1)$, C_2 is the upper part of the circle $(x+1)^2 + (y-1)^2 = 1$, and C_3 is the part of the ellipse $\frac{x^2}{4} + y^2 = 1$ that satisfies $x \ge 0$.

(c) $\displaystyle\int_C (x\ln(x^2+y^2)\,\mathbf{i} + y\ln(x^2+y^2)\,\mathbf{j}) \cdot d\mathbf{r}$,

where C is the boundary of the square with vertices $(-1,1)$, $(-1,-1)$, $(1,-1)$, and $(1,1)$.

Question 9. True or false? Justify your answer.

(a) If $\mathbf{F}$ is a conservative vector field, then $\mathbf{F} = \nabla f$ for a unique function f.

(b) If $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of paths, then $\mathbf{F}$ is conservative.

(c) If $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j}$ is such that P and Q have continuous partial derivatives on a connected region D and $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ in D, then F is conservative.

Question 10. Make sure that:

(a) You know the statements and proofs of the important theoretical results established in class.

(b) You know the important definitions given in class.

(c) You are able to solve problems in a timely manner. For this, it is important that you find the best way of solving each problem. A single problem can be solved by different methods; make sure that you are able to identify the most concise approach. Each problem[1] in this practice test is an exam-like question that could have been asked in the actual test, thus you should be able to solve it without excessive calculations. If your calculations are too long or you are spending too much time in a given problem, you are probably taking the wrong approach.

[1]By a problem, I mean a single unit of each question. For instance, item (a) in question 4 is an exam-like question, but the whole of question 4, with items (a) to (d), is not, since that would be too long for an exam. Also, it is unlikely that you will be asked to rewrite an iterated integral in all possible different ways, such as in question 1, as that would again take too long, but you may be asked to rewrite it in one or two different ways.