VANDERBILT UNIVERSITY

MATH 2300 – MULTIVARIABLE CALCULUS

Practice Test 2

Directions. This practice test should be used as a study guide, illustrating the concepts that will be emphasized in the test. This does not mean that the actual test will be restricted to the content of the practice. Try to identify, from the questions below, the concepts and sections that you should master for the test. For each question in the practice test, study the ideas and techniques connected to the problem, even if they are not directly used in your solution.

Take this also as an opportunity to practice how you will write your solutions in the test. For this, write clearly, legibly, and in a logical fashion. Make precise statements (for instance, write an equal sign if two expressions are equal; say that one expression is a consequence of another when this is the case, etc).

Question 1. Express $\iiint_D f(x,y,z)\,dV$ as an iterated integral in six different ways, where D is the solid bounded by the given surfaces.

(a) $y = x^2$, $z = 0$, $y + 2z = 4$.

(b) $x = 2$, $y = 2$, $z = 0$, $x + y - 2z = 2$.

Solution 1. (a) The region D is shown in figure 1.

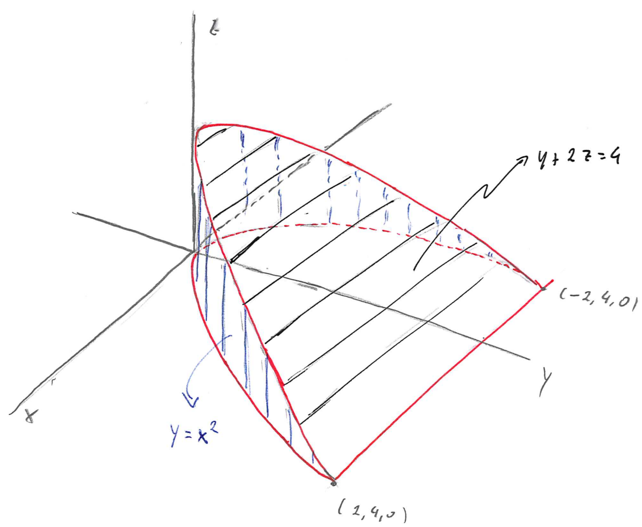


FIGURE 1. Region of question 1(a).

For each order of integration, we will write the region D in the form

$$D = \{ \text{ last integral}, \ \text{ second integral}, \ \text{ first integral } \},$$

meaning that the limits are presented in the order of the last integration to the first. From the picture, we obtain

$$\begin{aligned} D &= \{-2 \le x \le 2, x^2 \le y \le 4, 0 \le z \le 2 - \frac{y}{2}\} \\ &= \{0 \le y \le 4, -\sqrt{y} \le x \le \sqrt{y}, 0 \le z \le 2 - \frac{y}{2}\} \\ &= \{0 \le y \le 4, 0 \le z \le 2 - \frac{y}{2}, -\sqrt{y} \le x \le \sqrt{y}\} \\ &= \{0 \le z \le 2, 0 \le y \le 4 - 2z, -\sqrt{y} \le x \le \sqrt{y}\} \\ &= \{-2 \le x \le 2, 0 \le z \le 2 - \frac{x^2}{2}, x^2 \le y \le 4 - 2z\} \\ &= \{0 \le z \le 2, -\sqrt{4-2z} \le x \le \sqrt{4-2z}, x^2 \le y \le 4 - 2z\}. \end{aligned}$$

Above, to determine the limits of the x and z variables in terms of each other (the last two lines), we proceeded as follows. The intersection of $y = x^2$ and $y + 2z = 4$ corresponds to the upper red curve in figure 1. The projection of this curve into the xz-plane is obtained upon plugging $y = x^2$ into $y + 2z = 4$, which gives $x^2 + 2z = 4$.

Writing the integral according to each one of the forms of D expressed above, we find

$$\begin{aligned}
\iiint_D f(x,y,z)\,dV &= \int_{-2}^{2}\int_{x^2}^{4}\int_{0}^{2-\frac{y}{2}} f(x,y,z)\,dz\,dy\,dx \\
&= \int_{0}^{4}\int_{-\sqrt{y}}^{\sqrt{y}}\int_{0}^{2-\frac{y}{2}} f(x,y,z)\,dz\,dx\,dy \\
&= \int_{0}^{4}\int_{0}^{2-\frac{y}{2}}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y,z)\,dx\,dz\,dy \\
&= \int_{0}^{2}\int_{0}^{4-2z}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y,z)\,dx\,dy\,dz \\
&= \int_{-2}^{2}\int_{0}^{2-\frac{x^2}{2}}\int_{x^2}^{4-2z} f(x,y,z)\,dy\,dz\,dx \\
&= \int_{0}^{2}\int_{-\sqrt{4-2z}}^{\sqrt{4-2z}}\int_{x^2}^{4-2z} f(x,y,z)\,dy\,dx\,dz.
\end{aligned}$$

(b) The region D is shown in figure 2.

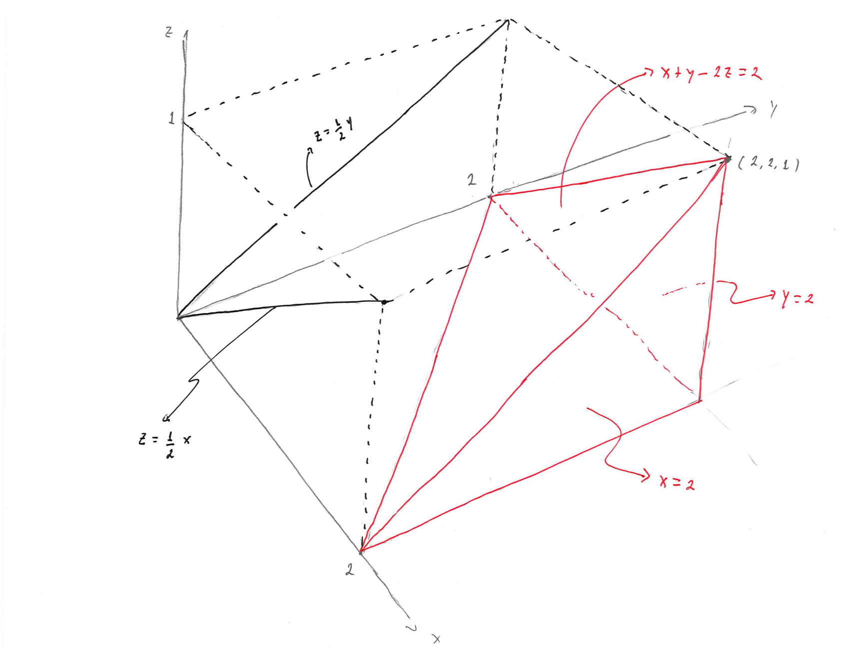


FIGURE 2. Region of question 1(b).

The lines $z = \frac{x}{2}$ and $z = \frac{y}{2}$ correspond to the intersections of the plane $x + y - 2z = 2$ with the planes $y = 2$ and $x = 2$, respectively. From the picture we find

$$\begin{aligned} D &= \{0 \le x \le 2, 2 - x \le y \le 2, 0 \le z \le \frac{1}{2}(x + y - 2)\} \\ &= \{0 \le y \le 2, 2 - y \le x \le 2, 0 \le z \le \frac{1}{2}(x + y - 2)\} \\ &= \{0 \le y \le 2, 0 \le z \le \frac{y}{2}, 2 - y + 2z \le x \le 2\} \\ &= \{0 \le z \le 1, 2z \le y \le 2, 2 - y + 2z \le x \le 2\} \\ &= \{0 \le x \le 2, 0 \le z \le \frac{x}{2}, 2 - x + 2z \le y \le 2\} \\ &= \{0 \le z \le 1, 2z \le x \le 2, 2 - x + 2z \le y \le 2\}. \end{aligned}$$

The integrals can now be easily written following the orders described in the above six representations of region D, as in part (a).

Question 2. Rewrite the integral

$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x,y,z)\, dy\, dz\, dx$$

as an equivalent iterated integral in the five other orders.

Solution 2. The region of integration, D, is shown in figure 3.

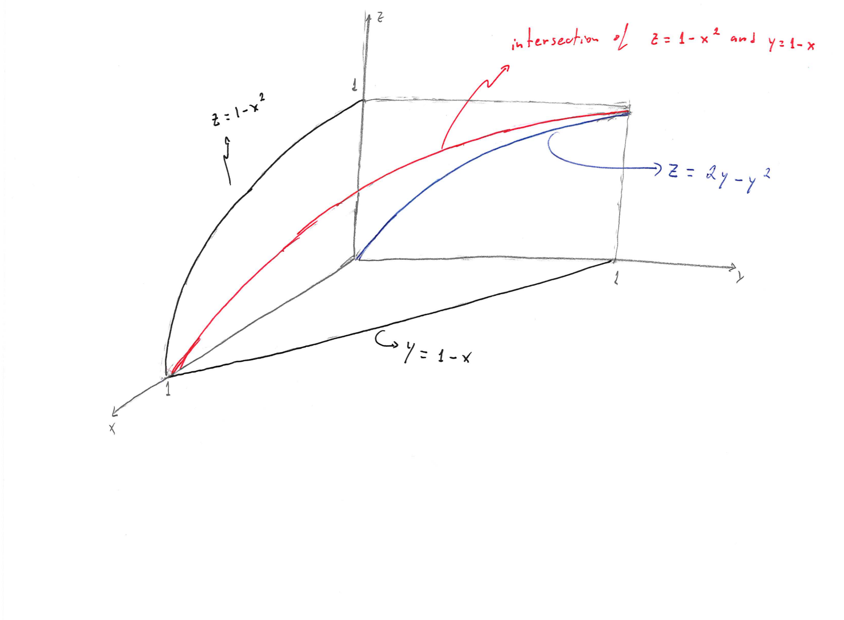


FIGURE 3. Region of question 2.

The blue curve shown in figure 3, i.e., $z = 2y - y^2$, corresponds to the projection onto the yz-plane of the intersection of $z = 1 - x^2$ and $y = 1 - x$ (which is depicted in red in the figure). It was found upon using $x = 1 - y$ into $z = 1 - x^2$.

From the picture, we obtain the following descriptions of D:

$$\begin{aligned} D &= \{0 \le x \le 1, 0 \le z \le 1 - x^2, 0 \le y \le 1 - x\} \\ &= \{0 \le z \le 1, 0 \le x \le \sqrt{1-z}, 0 \le y \le 1 - x\} \\ &= \{0 \le y \le 1, 0 \le x \le 1 - y, 0 \le z \le 1 - x^2\} \\ &= \{0 \le x \le 1, 0 \le y \le 1 - x, 0 \le z \le 1 - x^2\}. \end{aligned}$$

The description of the region when we integrate first in x requires a little more care as it has to be split into two regions. This is because when x varies inside D, in the region above $z = 2y - y^2$ (blue curve), x varies from zero to the cylinder $z = 1 - x^2$. But in the region below the curve $z = 2y - y^2$ (blue curve), x varies from zero to the plane $y = 1 - x$. Thus

$$\begin{aligned} D = &\{0 \le y \le 1, 0 \le z \le 2y - y^2, 0 \le x \le 1 - y\} \\ &\cup \{0 \le y \le 1, 2y - y^2 \le z \le 1, 0 \le x \le \sqrt{1-z}\}, \end{aligned}$$

and

$$D = \{0 \le z \le 1, 0 \le y \le 1 - \sqrt{1-z}, 0 \le x \le \sqrt{1-z}\}$$
$$\cup \{0 \le z \le 1, \le 1 - \sqrt{1-z} \le y \le 1, 0 \le x \le 1-y\},$$

where $y \le 1 - \sqrt{1-z}$ comes from solving for y in $z = 2y - y^2$ since

$$z = 2y - y^2 \Leftrightarrow y^2 - 2y + 1 = 1 - z \Leftrightarrow (y-1)^2 = 1 - z.$$

We choose the negative root because it corresponds to $1 - y = x = \sqrt{1-z}$.

The integrals are now easily written from the descriptions of D given above. Let us write them for the last two cases:

$$\begin{aligned}\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x,y,z)\,dy\,dz\,dx &= \int_0^1 \int_0^{2y-y^2} \int_0^{1-y} f(x,y,z)\,dx\,dz\,dy \\ &+ \int_0^1 \int_{2y-y^2}^1 \int_0^{\sqrt{1-z}} f(x,y,z)\,dx\,dz\,dy \\ &= \int_0^1 \int_0^{1-\sqrt{1-z}} \int_0^{\sqrt{1-z}} f(x,y,z)\,dx\,dy\,dz \\ &+ \int_0^1 \int_{1-\sqrt{1-z}}^1 \int_0^{1-y} f(x,y,z)\,dx\,dx\,dz.\end{aligned}$$

Question 3. (a) Write $\iiint\limits_D f(x,y,z)\,dV$ as an iterated integral in Cartesian coordinates in two different ways, where D is the region bounded by $z+x^2+y^2=1$, $y+x=0$, and the xy-plane.
(b) Using Cartesian, cylindrical, or spherical coordinates, evaluate the integral in (a) if $f(x,y,z) = \sqrt{x^2+y^2}$.

Solution 3. (a) The region of integration, D, is shown in figure 4. The intersection of $z+x^2+y^2=1$

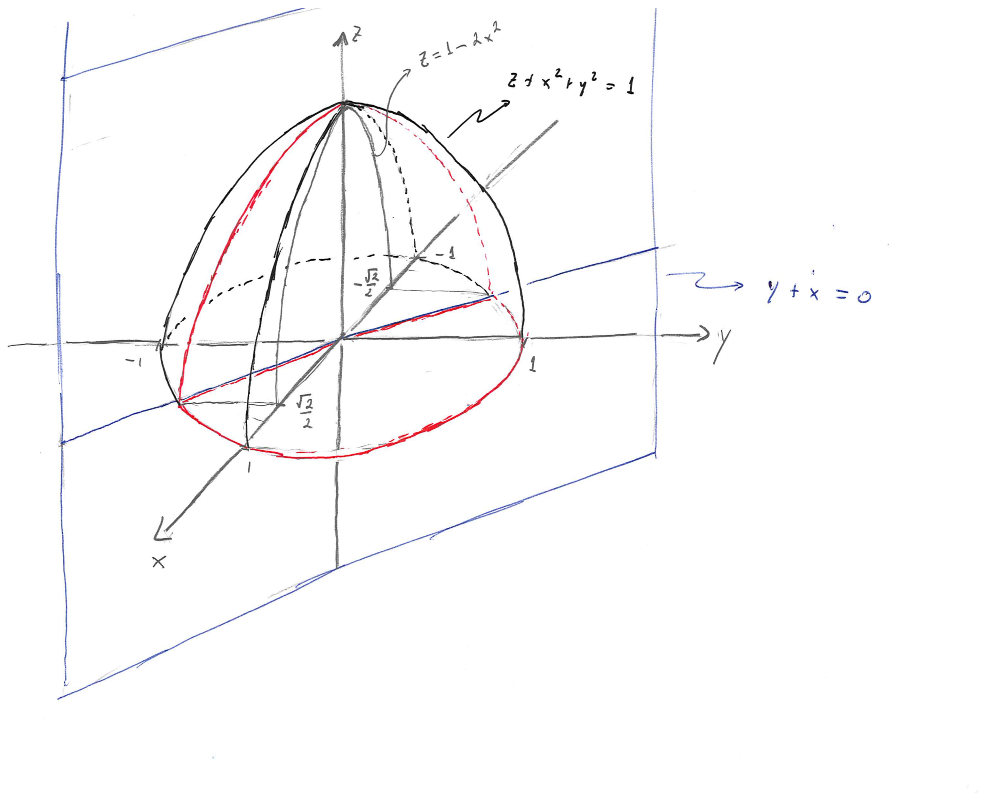


FIGURE 4. Region of question 3.

with $y+x=0$ gives $z=1-2x^2$. When $z=0$ this produces $x=\pm\frac{\sqrt{2}}{2}$.

Let us first integrate in the order $dz\,dy\,dx$. From the picture, we see that y varies between $-x$ and $\sqrt{1-x^2}$ when $-\frac{\sqrt{2}}{2} \le x \le \frac{\sqrt{2}}{2}$ and between $-\sqrt{1-x^2}$ and $\sqrt{1-x^2}$ when $\frac{\sqrt{2}}{2} \le x \le 1$. Thus,

$$\iiint\limits_D f(x,y,z)\,dV = \int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \int_{-x}^{\sqrt{1-x^2}} \int_0^{1-x^2-y^2} f(x,y,z)\,dz\,dy\,dx$$
$$+ \int_{\frac{\sqrt{2}}{2}}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^{1-x^2-y^2} f(x,y,z)\,dz\,dy\,dx.$$

Next, let us integrate in the order $dx\,dz\,dy$. Again, the integral has to be split: x varies between the plane and the paraboloid when $-\frac{\sqrt{2}}{2} \le y \le \frac{\sqrt{2}}{2}$, and between two sides of the paraboloid when

$\frac{\sqrt{2}}{2} \leq y \leq 1$. We find

$$\iiint\limits_D f(x,y,z)\,dV = \int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{1-z-y^2}} f(x,y,z)\,dx\,dz\,dy$$

$$+ \int_{\frac{\sqrt{2}}{2}}^{1} \int_0^{\sqrt{1-y^2}} \int_{-\sqrt{1-z-y^2}}^{\sqrt{1-z-y^2}} f(x,y,z)\,dx\,dz\,dy.$$

(b) In cylindrical coordinates, the paraboloid reads $z = 1 - r^2$. The line $x + y = 0$ becomes $r\sin\theta + r\cos\theta = 0$, thus, $\tan\theta = -1$, hence $\theta = -\frac{\pi}{4}$ and $\theta = \frac{3\pi}{4}$. Then

$$\iiint\limits_D f(x,y,z)\,dV = \int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} \int_0^1 \int_0^{1-r^2} r^2\,dz\,dr\,d\theta = \frac{2\pi}{15}.$$

Remark. Strictly speaking we should split the integral using $0 \leq \theta \leq \frac{3\pi}{4}$ and $\frac{7\pi}{4} \leq \theta \leq 2\pi$, since the domain of θ is $[0, 2\pi]$. But we can verify that this will not change the value of the above integral.

Question 4. A region R in the xy-plane is given. Find equations for a transformation T that maps a rectangular region S in the uv-plane onto R, where the sides of S are parallel to the u- and v-axes.

(a) R is bounded by $y = -x - 3$, $y = -5$, $y + x + 1 = 0$, and the x-axis.

(b) R is bounded by $y = 1/x$, $y = 4/x$, and the lines $y = x$ and $y = 4x$ in the first quadrant.

(c) R is bounded $y = \sin x + 2$, $x = 0$, $x = 2\pi$, and $y = \sin x$.

(d) R is bounded $x^2 + y^2 = 9$, $x^2 + y^2 = 1$, $y = x$, and $y = -x$ (this actually determines more than one region; choose one).

Solution 4. The four regions in (a)-(d) are shown in figure 5.

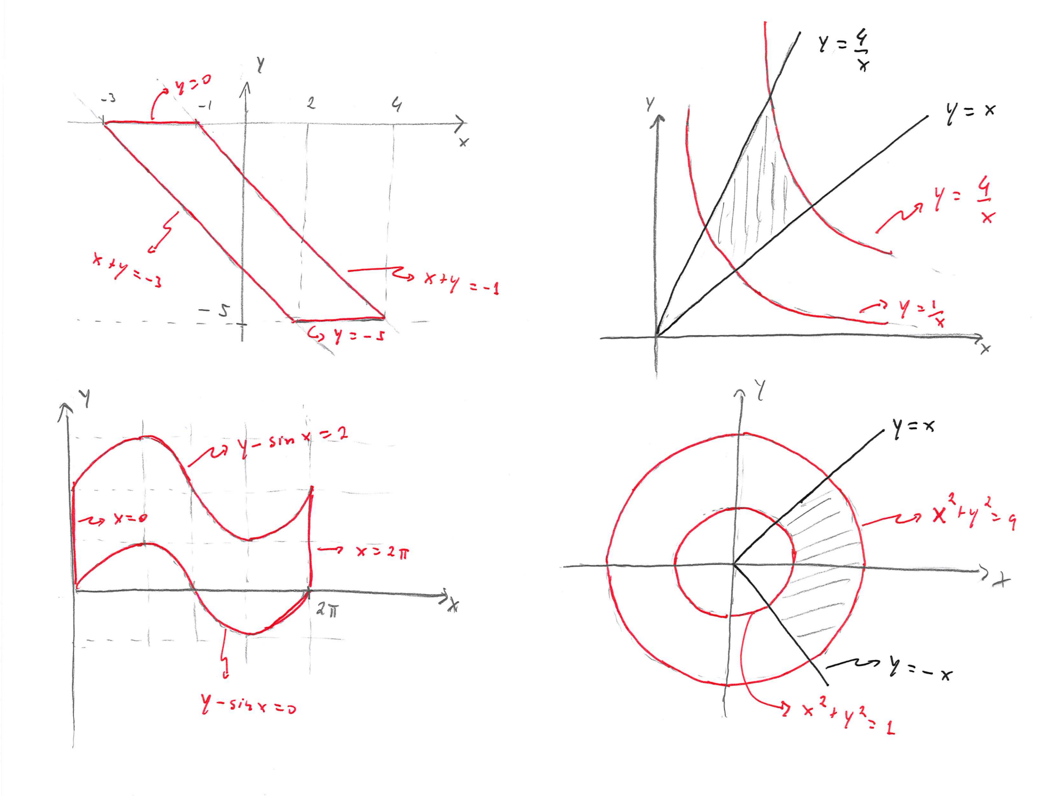


FIGURE 5. Regions of question 4.

(a) Set $u = x + y$ and $v = y$, so that $x = u - v$ and $y = v$. The region on the uv-plane is $-3 \leq u \leq -1$, $-5 \leq v \leq 0$.

(b) Set $u = xy$ and $v = \frac{y}{x}$, so the curves correspond to $u = 1$, $u = 4$, $v = 1$, and $v = 4$. Then $x = \sqrt{\frac{u}{v}}$ and $y = \sqrt{uv}$. The region on the uv-plane is $1 \leq u \leq 4$, $1 \leq v \leq 4$.

(c) Set $u = \frac{x}{2\pi}$ and $v = y - \sin x$, so that $x = 2\pi u$ and $y = v + \sin(2\pi u)$. The region on the uv-plane is $0 \leq u \leq 1$, $1 \leq v \leq 2$.

(d) Using $x = u \cos v$ and $y = u \sin v$, we have $1 \leq u \leq 3$, $-\frac{\pi}{4} \leq v \leq \frac{\pi}{4}$.

Question 5. Evaluate the integrals:

(a) $$\iint\limits_D \frac{\sqrt{x^2+16y^2+8xy}}{2\sqrt{x^2+4y^2}}\,dA,$$

where D is the region bounded by $x^2+4y^2=4$, $x^2+4y^2=16$, $y-x-1=0$, and $y-x+2=0$, and $x\geq 0$.

(b) $$\iint\limits_D (1-2y)\,dA,$$

where D is the region bounded by the curve $x+y^2=4$ and the line joining the points $(-5,-3)$ and $(0,2)$.

Solution 5. (a) The region D is shown in figure 6. We will make a change of variables to simplify

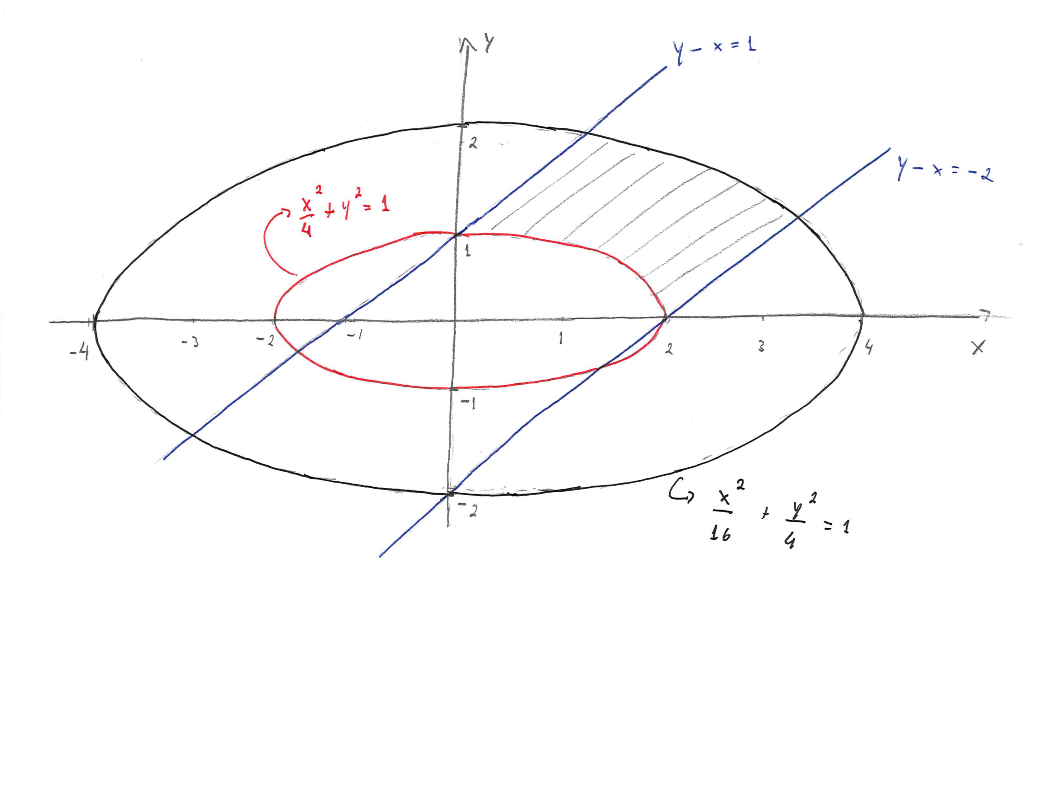


FIGURE 6. Regions of question 5(a).

it. Set $u^2=\frac{x^2}{4}+y^2$, $u\geq 0$, so that the inner ellipse (red curve) corresponds to $u=1$ and the outer ellipse (black curve) to $u=2$. Set $v=y-x$, so that the two blue lines correspond to $v=1$ and $v=-2$.

Inverting the equations for $x=x(u,v)$ and $y=y(u,v)$ we find

$$x=\frac{2}{5}(-2v+\sqrt{5u^2-v^2}),$$

and

$$y = \frac{v}{5} + \frac{2}{5}\sqrt{5u^2 - v^2}.$$

One can verify that these expressions give a well-defined transformation for $1 \le u \le 2$ and $-2 \le v \le 1$. The Jacobian of the change of variables is

$$\frac{\partial(x,y)}{\partial(u,v)} = \frac{2u}{\sqrt{5u^2 - v^2}}.$$

On the other hand,

$$f(x(u,v), y(u,v)) = \frac{\sqrt{5u^2 - v^2}}{2u},$$

thus

$$\iint\limits_D \frac{\sqrt{x^2 + 16y^2 + 8xy}}{2\sqrt{x^2 + 4y^2}}\, dA = \int_{-2}^{1}\int_{1}^{2} \frac{\sqrt{5u^2 - v^2}}{2u} \frac{2u}{\sqrt{5u^2 - v^2}}\, du\, dv = 3.$$

(b) The region is shown in figure 7 (upper left picture).

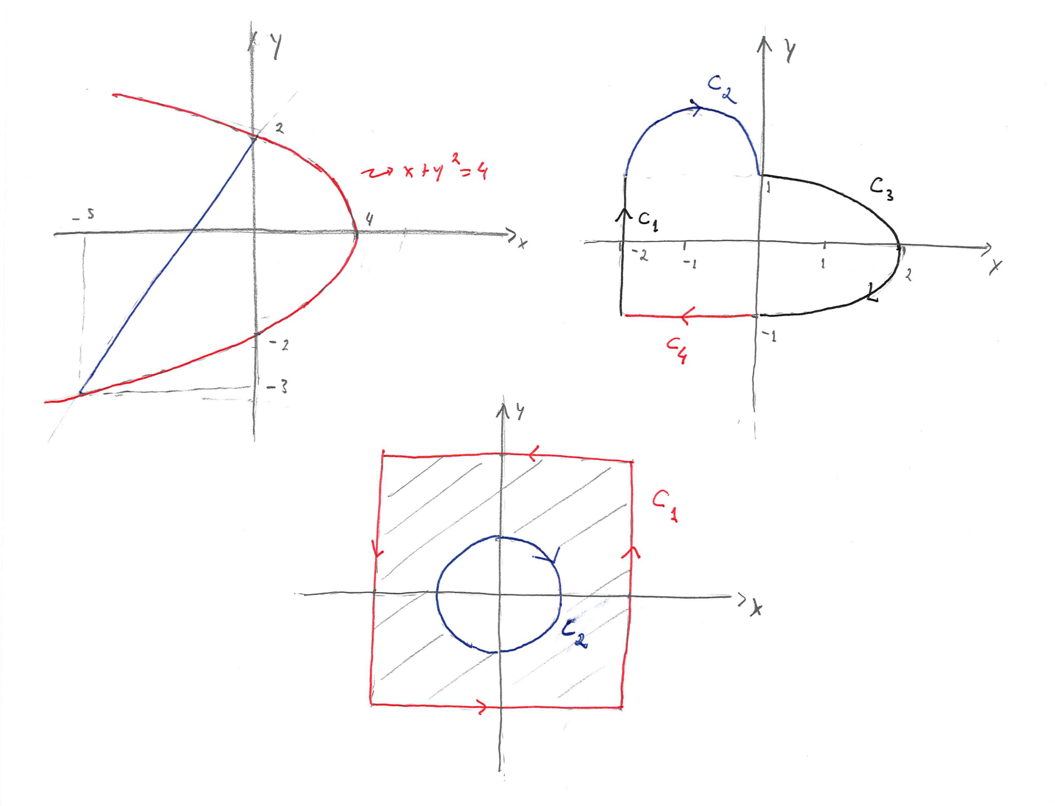


FIGURE 7. Regions of questions 5(b), 8(b), and 8(c).

Using Green's theorem,

$$\iint\limits_D (1-2y)\,dA = \iint\limits_D \left(\frac{\partial x}{\partial x} - \frac{\partial y^2}{\partial y}\right) dA$$
$$= \int_{\partial D} (y^2\,dx + x\,dy)$$
$$= \int_{C_1} (y^2\,dx + x\,dy) - \int_{C_2} (y^2\,dx + x\,dy),$$

where C_1 is the curve $x + y^2 = 4$ (red curve) oriented counter-clockwise and C_2 is the line joining the points $(-5,-3)$ and $(0,2)$ (blue curve) oriented clockwise.We can parametrize C_1 by $x = 4-y^2$, $y = y$, $-3 \le y \le 2$, and C_2 by $x = 5t-5$, $y = 5t-3$, $0 \le t \le 1$. Then we easily compute

$$\int_{C_1} (y^2\,dx + x\,dy) = \frac{245}{6} \text{ and } \int_{C_2} (y^2\,dx + x\,dy) = -\frac{5}{6},$$

so that $\iint\limits_D (1-2y)\,dA = \frac{245}{6} + \frac{5}{6} = \frac{125}{3}$.

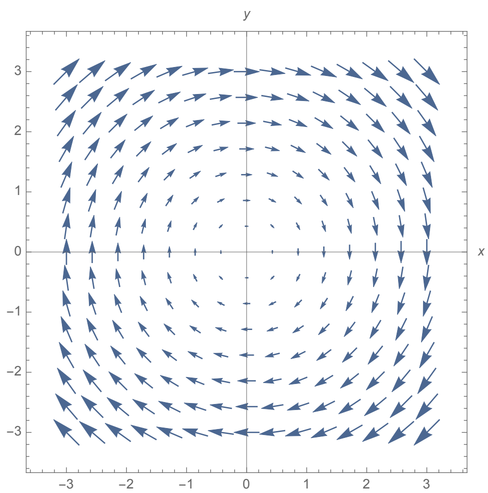
Question 6. Match the vector fields $\mathbf{F}$ with the given plots.

(a) $\mathbf{F}(x, y) = \cos x\,\mathbf{i} - \mathbf{j}$.

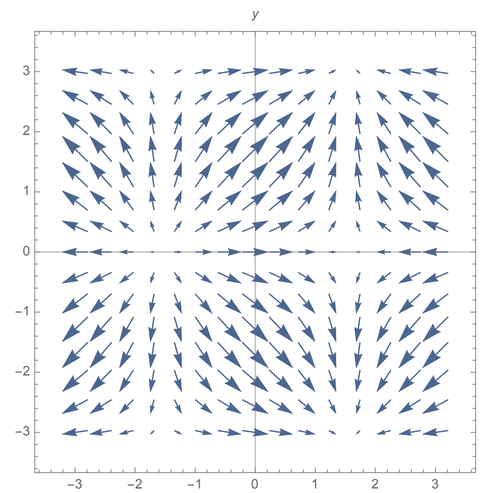
(b) $\mathbf{F}(x, y) = \frac{x}{2}\,\mathbf{i} + y\,\mathbf{j}$.

(c) $\mathbf{F}(x, y) = y\,\mathbf{i} - x\,\mathbf{j}$.

(d) $\mathbf{F}(x, y) = \cos x\,\mathbf{i} + \sin y\,\mathbf{j}$.



(I)



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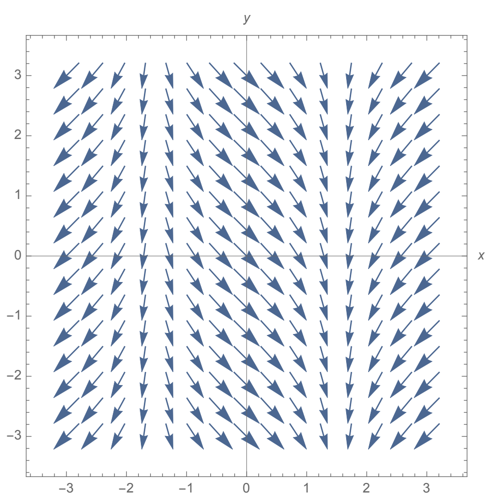

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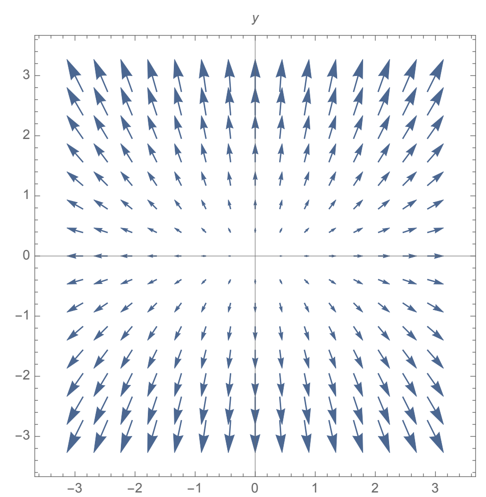

(IV)

Solution 6. (a) Constant y component pointing downward: III. (b) Similar to $\mathbf{F}(\mathbf{x}) = \mathbf{x}$ studied in class, but tilted by half in the x component: IV. (c) Rotation about the origin (similarly to what was studied in class): I. (d) Periodic in both the x and y components: II.

Question 7. Determine whether or not the vector field $\mathbf{F}$ is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$.

(a) $\mathbf{F}(x, y) = (y^2 - 2x)\,\mathbf{i} + 2xy\,\mathbf{j}$.

(b) $\mathbf{F}(x, y) = \cos y\,\mathbf{i} + x \sin y\,\mathbf{j}$.

Solution 7. (a) $P(x, y) = y^2 - 2x$, $Q(x, y) = 2xy$, $\frac{\partial Q(x,y)}{\partial x} = 2y = \frac{\partial P(x,y)}{\partial y}$. The domain of $\mathbf{F}$ is $\mathbb{R}^2$, which is simply connected, thus $\mathbf{F}$ is conservative: $\mathbf{F} = \nabla f$, so $\frac{\partial f}{\partial x} = P$ and $\frac{\partial P}{\partial y} = Q$. Hence

$$\frac{\partial f(x, y)}{\partial x} = y^2 - 2x \Rightarrow f(x, y) = xy^2 - x^2 + g(y).$$

Next

$$\frac{\partial f(x, y)}{\partial y} = 2xy + g'(y) = Q(x, y) = 2xy \Rightarrow g'(y) = 0.$$

Thus g is equal to a constant, which we can assume to be zero. Thus $f(x, y) = xy^2 - x^2$.

(b) $P(x, y) = \cos y$, $Q(x, y) = x \sin y$, $\frac{\partial Q(x,y)}{\partial x} = \sin y \neq -\sin y = \frac{\partial P(x,y)}{\partial y}$, hence the vector field is not conservative.

Question 8. Evaluate the line integral, where C is the given curve.

(a) $\displaystyle\int_C \frac{x}{y}\,ds$, where C is given by $x = t^3$, $y = t^4$, $1 \le t \le 2$.

(b) $\displaystyle\int_C (xy\,\mathbf{i} + \frac{x^2}{2}\,\mathbf{j}) \cdot d\mathbf{r}$.

The curve C is given by $C = C_1 \cup C_2 \cup C_3$, where C_1 is the line segment joining $(-2,-1)$ to $(-2,1)$, C_2 is the upper part of the circle $(x+1)^2 + (y-1)^2 = 1$, and C_3 is the part of the ellipse $\frac{x^2}{4} + y^2 = 1$ that satisfies $x \ge 0$.

(c) $\displaystyle\int_C (x\ln(x^2+y^2)\,\mathbf{i} + y\ln(x^2+y^2)\,\mathbf{j}) \cdot d\mathbf{r}$,

where C is the boundary of the square with vertices $(-1,1)$, $(-1,-1)$, $(1,-1)$, and $(1,1)$.

Solution 8. (a) Using the parametrization we find

$$\int_C \frac{x}{y}\,ds = \int_1^2 \frac{t^3}{t^4}\sqrt{(3t^2)^2 + (4t^3)^2}\,dt = \int_1^2 t\sqrt{9+16t^2}\,dt = \frac{1}{48}(73\sqrt{73} - 125).$$

(b) The curve $C_1 \cup C_2 \cup C_3$ is shown in figure 7 (upper right picture). We are integrating the vector field $\mathbf{F}(x,y) = xy\mathbf{i} + \frac{x^2}{2}\mathbf{j}$ which we easily verify to be conservative.

Let C_4 be the line segment joining $(0,-1)$ to $(-2,-1)$, as as shown in figure 7. The region enclosed by the curve $C_1 \cup C_2 \cup C_3 \cup C_4$ is simply connected, thus

$$\int_{C_1\cup C_2\cup C_3\cup C_4} (xy\,\mathbf{i} + \frac{x^2}{2}\,\mathbf{j}) \cdot d\mathbf{r} = 0,$$

hence

$$\begin{aligned}\int_{C_1\cup C_2\cup C_3} (xy\,\mathbf{i} + \frac{x^2}{2}\,\mathbf{j}) \cdot d\mathbf{r} &= -\int_{C_4} (xy\,\mathbf{i} + \frac{x^2}{2}\,\mathbf{j}) \cdot d\mathbf{r} \\ &= -\int_{C_4} (xy\,dx + \frac{x^2}{2}\,dy) \\ &= \int_0^{-2} x\,dx = 2,\end{aligned}$$

where we used that along C_4 we have $y = -1$ and $dy = 0$.

Alternative solution. Since the vector field is conservative, we know that there exists f such that $\mathbf{F} = \nabla f$. Arguing as in question 7(a), we obtain that $f(x,y) = \frac{x^2 y}{2}$. The fundamental theorem of line integrals then gives

$$\int_{C_1\cup C_2\cup C_3} (xy\,\mathbf{i} + \frac{x^2}{2}\,\mathbf{j}) \cdot d\mathbf{r} = f(0,-1) - f(-2,1) = 0 - \frac{4\cdot(-1)}{2} = 2.$$

(c) Denote the given curve by C_1 and orient it counter-clockwise. Write the integral as $\int_{C_1}(P\,dx + Q\,dy)$, where $P(x,y) = x\ln(x^2+y^2)$ and $Q(x,y) = y\ln(x^2+y^2)$. We have $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$, however, we cannot invoke Green's theorem in the region enclosed by C_1 because these partial derivatives are not continuous at the origin. However, Green's theorem does apply in the region D bounded by C_1

and C_2, where C_2 is the circle of radius $\frac{1}{2}$ about the origin oriented clockwise; see figure 7 (bottom picture). Then

$$0 = \iint\limits_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \int_{\partial D} \mathbf{F} \cdot d\mathbf{r} = \int_{C_1} \mathbf{F} \cdot d\mathbf{r} + \int_{C_2} \mathbf{F} \cdot d\mathbf{r}.$$

Using the parametrization $x = \frac{1}{2}\cos t$ and $y = \frac{1}{2}\sin t$, we easily find that $\int_{C_2} \mathbf{F} \cdot d\mathbf{r} = 0$, thus $\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = 0$.

Question 9. True or false? Justify your answer.

(a) If $\mathbf{F}$ is a conservative vector field, then $\mathbf{F} = \nabla f$ for a unique function f.

(b) If $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of paths, then $\mathbf{F}$ is conservative.

(c) If $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j}$ is such that P and Q have continuous partial derivatives on a connected region D and $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ in D, then F is conservative.

Solution 9. (a) False, f is not unique: if f is a potential function, so is $f + cte$. (b) True, assuming that the region is connected (the statement was not very precise about the region where the vector field is defined). (c) False, the correct assumption is that the region be simply connected.

Question 10. Make sure that:

(a) You know the statements and proofs of the important theoretical results established in class.

(b) You know the important definitions given in class.

(c) You are able to solve problems in a timely manner. For this, it is important that you find the best way of solving each problem. A single problem can be solved by different methods; make sure that you are able to identify the most concise approach. Each problem[1] in this practice test is an exam-like question that could have been asked in the actual test, thus you should be able to solve it without excessive calculations. If your calculations are too long or you are spending too much time in a given problem, you are probably taking the wrong approach.

[1] By a problem, I mean a single unit of each question. For instance, item (a) in question 4 is an exam-like question, but the whole of question 4, with items (a) to (d), is not, since that would be too long for an exam. Also, it is unlikely that you will be asked to rewrite an iterated integral in all possible different ways, such as in question 1, as that would again take too long, but you may be asked to rewrite it in one or two different ways.