

VANDERBILT UNIVERSITY

MATH 2300 – MULTIVARIABLE CALCULUS

*Practice Test 1*

**Directions.** This practice test should be used as a study guide, illustrating the concepts that will be emphasized in the test. This does not mean that the actual test will be restricted to the content of the practice. Try to identify, from the questions below, the concepts and sections that you should master for the test. For each question in the practice test, study the ideas and techniques connected to the problem, even if they are not directly used in your solution.

Take this also as an opportunity to practice how you will write your solutions in the test. For this, write clearly, legibly, and in a logical fashion. Make precise statements (for instance, write an equal sign if two expressions are equal; say that one expression is a consequence of another when this is the case, etc).

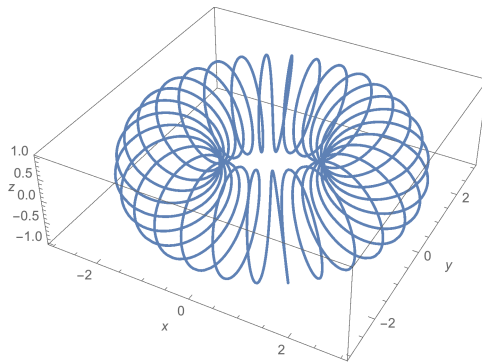
**Question 1.** Match the curves below with the given graphs. Justify your answer.

(a)  $\mathbf{r}(t) = \langle (2 + \sin(5t)) \cos t, (2 + \sin(5t)) \sin t, \cos(5t) \rangle$

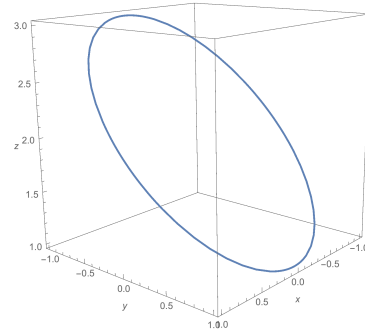
(b)  $\mathbf{r}(t) = \langle \cos t, \sin t, 2 - \sin t \rangle$

(c)  $\mathbf{r}(t) = \langle \cos^2 t, \sin^2 t, \frac{1}{2}t \rangle$

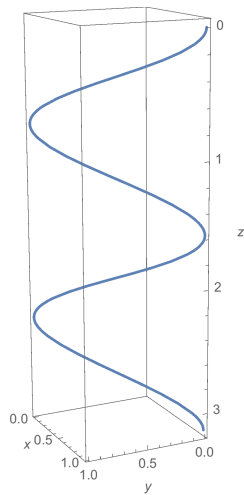
(d)  $\mathbf{r}(t) = \langle \cos t, \sin t, \cos(2t) \rangle$



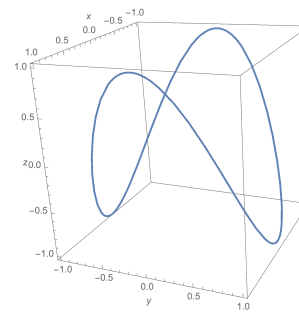
(I)



(II)



(III)



(IV)

**Question 2.** Where does the line through the points  $(-3, 1, 0)$  and  $(-1, 5, 6)$  intersect the plane  $2x + y - z + 2 = 0$ ?

**Question 3.** Calculate the limit of the given functions, or show that the limit does not exist.

(a)  $\lim_{(x,y) \rightarrow (0,0)} \frac{5x^2y}{x^2 + 2y^2}$ .

(b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy - y}{(x - 1)^2 + y^2}$ .

**Question 4.**

(a) Let  $F$  be a differentiable function of two variables  $x$  and  $y$ , and suppose that  $F(x, y) = 5$  defines  $y$  implicitly as a differentiable function of  $x$ . Show that

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} \frac{dy}{dx} = 0.$$

(b) State the implicit function theorem. Explain, in words, why it is true. (*Hint:* use pictures to illustrate your argument.)

**Question 5.** Consider the function

$$f(x, y) = 4xy^2 - x^2y^2 - xy^3.$$

Let  $D$  be the closed triangular region in the  $xy$ -plane with vertices  $(0, 0)$ ,  $(0, 6)$ , and  $(6, 0)$ .

- (a) Explain why  $f$  has an absolute maximum value and an absolute minimum value in  $D$ .
- (b) Find the absolute maximum value and the absolute minimum value of  $f$  in  $D$ .

**Question 6.** True or false? Justify your answer.

(a) If  $f(x, y) \rightarrow L$  as  $(x, y) \rightarrow (a, b)$  along every straight line through  $(a, b)$  then

$$\lim_{(x,y) \rightarrow (a,b)} f(x, y) = L.$$

(b) For any vectors  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$ ,  $(\mathbf{u} + \mathbf{v}) \times \mathbf{w} = \mathbf{u} \times \mathbf{w} + \mathbf{v} \times \mathbf{w}$ .

(c) If  $\mathbf{r} : (a, b) \rightarrow \mathbb{R}^3$  is differentiable, and  $|\mathbf{r}(t)| = 3$  for all  $t \in (a, b)$ , then  $\mathbf{r}'(t)$  is orthogonal to  $\mathbf{r}(t)$  for all  $t \in (a, b)$ .