VANDERBILT UNIVERSITY

MATH 2300 – MULTIVARIABLE CALCULUS

Examples of section 15.9

Question 1. Evaluate

$$\iint\limits_R e^{x+y}\,dA,$$

where D is given by $|x|+|y|\leq 1$.

Solution 1. The region D is the square in the xy-plane with vertices $(1,0)$, $(0,1)$, $(-1,0)$, and $(0,-1)$. Let $u=x+y$ and $v=-x+y$. Then $u+v=2y$, $u-v=2x$, and therefore

$$x=\frac{1}{2}(u-v),$$

and

$$y=\frac{1}{2}(u+v).$$

We see that

$$|u|=|x+y|\leq |x|+|y|\leq 1 \Rightarrow -1\leq u\leq 1,$$

and

$$|v|=|-x+y|\leq |x|+|y|\leq 1 \Rightarrow -1\leq v\leq 1,$$

and we conclude that D is the image through the transformation $T(u,v)=(x,y)$ of the square with vertices $(1,1)$, $(1,-1)$, $(-1,-1)$, and $(-1,1)$ in the uv-plane.

Computing the Jacobian,

$$\frac{\partial(x,y)}{\partial(u,v)}=\det\begin{bmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{bmatrix}=\det\begin{bmatrix}\frac{1}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}=\frac{1}{2}.$$

Therefore

$$\iint\limits_R e^{x+y}\,dA=\frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}e^{u}\,dudv=e-e^{-1}.$$