VANDERBILT UNIVERSITY

MATH 2300 – MULTIVARIABLE CALCULUS

Examples of section 15.8

Question 1. Using spherical coordinates, evaluate

$$\iiint_D \sqrt{x^2+y^2+z^2}\,dV,$$

where D lies above the cone $z=\sqrt{x^2+y^2}$ and between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$.

Solution 1. In spherical coordinates, $z=\sqrt{x^2+y^2}$ is equivalent to $\phi=\frac{\pi}{4}$, and $\sqrt{x^2+y^2+z^2}=\rho$. The region D is then given by

$$D=\{(\rho,\theta,\phi)\,|\,1\le\rho\le 2, 0\le\theta\le 2\pi, 0\le\phi\le\frac{\pi}{4}\}.$$

Thus,

$$\begin{aligned}\iiint_D \sqrt{x^2+y^2+z^2}\,dV, &= \int_0^{\frac{\pi}{4}}\int_0^{2\pi}\int_1^2 \rho\rho^2\sin\phi\,d\rho\,d\theta\,d\phi\\ &= \int_0^{\frac{\pi}{4}}\sin\phi\,d\phi\int_0^{2\pi}d\theta\int_1^2\rho^3\,d\rho\\ &= \frac{15\pi}{2}(1-\frac{\sqrt{2}}{2}).\end{aligned}$$