VANDERBILT UNIVERSITY

MATH 2300 – MULTIVARIABLE CALCULUS

Examples of section 15.3

Question 1. Find the volume of the solid bounded by the paraboloids $z = 6 - x^2 - y^2$ and $z = 2x^2 + 2y^2$.

Solution 1. The paraboloids intersect when $6 - x^2 - y^2 = 2x^2 + 2y^2$, or when $x^2 + y^2 = 2$. For $x^2 + y^2 \leq 2$, the paraboloid $z = 6 - x^2 - y^2$ is above $z = 2x^2 + 2y^2$. Thus

$$V = \iint\limits_{x^2+y^2\leq 2} [(6 - x^2 - y^2) - (2x^2 + 2y^2)]\, dA$$
$$\iint\limits_{x^2+y^2\leq 2} [6 - 3(x^2 + y^2)]\, dA.$$

Since $x^2 + y^2 \leq 2$ corresponds to $r \leq \sqrt{2}$ polar coordinates, we find

$$\begin{aligned} V &= \int_0^{2\pi} \int_0^{\sqrt{2}} (6 - 3r^2) r\, dr d\theta \\ &= \int_0^{2\pi} d\theta \int_0^{\sqrt{2}} (6r - 3r^3)\, dr \\ &= 6\pi. \end{aligned}$$