VANDERBILT UNIVERSITY

MATH 2300 – MULTIVARIABLE CALCULUS

Examples of section 15.2

Question 1. Evaluate

$$\int\int_D x^2\,dA,$$

where D is the region inside the square with vertices $(1,1)$, $(-1,1)$, $(-1,-1)$, and $(1,-1)$, and outside the square with vertices $(0,1)$, $(-1,0)$, $(0,-1)$, $(1,0)$.

Solution 1. The region D can be written as $D = D_1 \cup D_2 \cup D_3 \cup D_4$, where

$$D_1 = \{(x,y)\,|\,0 \le x \le 1, -x+1 \le y \le 1\},$$

$$D_2 = \{(x,y)\,|\,-1 \le x \le 0, x+1 \le y \le 1\},$$

$$D_3 = \{(x,y)\,|\,0 \le x \le 1, -1 \le y \le x-1\},$$

and

$$D_4 = \{(x,y)\,|\,-1 \le x \le 0, -1 \le y \le -x-1\}.$$

Then

$$\begin{aligned}\int\int_D x^2\,dA &= \int_0^1\int_{1-x}^1 x^2\,dydx + \int_{-1}^0\int_{x+1}^1 x^2\,dydx \\ &\quad + \int_0^1\int_{-1}^{x-1} x^2\,dydx + \int_{-1}^0\int_{-1}^{-x-1} x^2\,dydx \\ &= 1.\end{aligned}$$