VANDERBILT UNIVERSITY

MATH 2300 – MULTIVARIABLE CALCULUS

Examples of section 14.6

Question 1. The temperature T in a metal ball is inversely proportional to the distance from the center of the ball, which we take to be the origin. The temperature at the point $(1, 2, 2)$ is $120°$.

(a) Find the rate of change of T at $(1, 2, 2)$ in the direction toward the point $(2, 1, 3)$.

(b) Show that at any point in the ball the direction of greatest increase in temperature is given by a vector that points toward the origin.

Solution 1. (a) The distance from (x, y, z) to the center is

$$\sqrt{x^2 + y^2 + z^2}$$

so that

$$T(x, y, z) = \frac{k}{\sqrt{x^2 + y^2 + z^2}},$$

where k is a constant. Since $T(1, 2, 2) = 120$ we have that $k = 360$, thus

$$T(x, y, z) = \frac{360}{\sqrt{x^2 + y^2 + z^2}}$$

A vector from $(1, 2, 2)$ to $(2, 1, 3)$ is $(2, 1, 3) - (1, 2, 2) = (1, -1, 1)$. Normalizing it we find

$$\mathbf{u} = \frac{1}{\sqrt{3}}\langle 1, -1, 1\rangle.$$

Computing the gradient, we find

$$\nabla T(x, y, z) = -360(x^2 + y^2 + z^2)^{-\frac{3}{2}}\langle x, y, z\rangle, \tag{1}$$

so that

$$\begin{aligned} D_{\mathbf{u}} &= \nabla T \cdot \mathbf{u} \\ &= -360(x^2 + y^2 + z^2)^{-\frac{3}{2}}\langle x, y, z\rangle \cdot \frac{1}{\sqrt{3}}\langle 1, -1, 1\rangle \\ &= -\frac{360}{\sqrt{3}}(x^2 + y^2 + z^2)^{-\frac{3}{2}}(x - y + z). \end{aligned}$$

Plugging in $(1, 2, 2)$ we find $D_{\mathbf{u}}T(1, 2, 2) = -\frac{40}{3\sqrt{3}}$.

(b) The direction of greatest increase is parallel to the gradient of T. From (1) we see that the gradient points in the direction opposite to $\langle x, y, z\rangle$, hence toward the origin.