

VANDERBILT UNIVERSITY

MATH 2300 – MULTIVARIABLE CALCULUS

Examples of section 14.5

Question 1. The radius of a right circular cone is increasing at a rate 1.8 in/s while its height is decreasing at a rate of 2.5 in/s . At what rate is the volume of the cone changing when the radius is 120 in and the height is 140 in ?

Solution 1. The volume is given by

$$V = \frac{1}{3}\pi r^2 h$$

where r is the radius and h the height. These are functions of the time t since they are changing over time. We want to find $\frac{dV}{dt}$. The chain rule gives

$$\frac{dV}{dt} = \frac{\partial V}{\partial r} \frac{dr}{dt} + \frac{\partial V}{\partial h} \frac{dh}{dt}.$$

Then

$$\frac{dV}{dt} = \frac{2}{3}\pi r h \frac{dr}{dt} + \frac{1}{3}\pi r^2 \frac{dh}{dt}.$$

We are given $\frac{dr}{dt} = 1.8$ and $\frac{dh}{dt} = -2.5$, so

$$\frac{dV}{dt} = \frac{2}{3} \times 1.8\pi r h + \frac{1}{3} \times (-2.5)\pi r^2.$$

When $r = 120$ and $h = 140$, we find $\frac{dV}{dt} = 8160\pi \text{ in}^3/\text{s}$.