VANDERBILT UNIVERSITY

MATH 2300 – MULTIVARIABLE CALCULUS

Examples of section 14.4

Question 1. Given that f is a differentiable function with $f(2,5) = 6$, $f_x(2,5) = 1$, and $f_y(2,5) = -1$, estimate $f(2.2, 4.9)$.

Solution 1. Notice that the function f itself is not given, but we can estimate $f(2.2, 4.9)$ using a linear approximation since $(2.2, 4.9)$ is near $(2, 5)$, and the value of f at $(2, 5)$ is given.

We have the linear approximation

$$\begin{aligned}
f(x, y) &\approx f(2,5) + f_x(2,5)(x - 2) + f_y(2,5)(y - 5) \\
&= 6 + 1(x - 2) + (-1)(y - 5) \\
&= x - y + 9.
\end{aligned}$$

Thus

$$\begin{aligned}
f(2.2, 4.9) &\approx 2.2 - 4.9 + 9 \\
&= 6.3.
\end{aligned}$$