VANDERBILT UNIVERSITY

MATH 2300 – MULTIVARIABLE CALCULUS

Examples of section 12.2

Question 1. Find $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - 2\mathbf{b}$ if $\mathbf{a} = \langle 1, 0, -1 \rangle$ and $\mathbf{b} = \langle 3, -2, 1 \rangle$.

Solution 1. It is immediate to compute

$$\mathbf{a} + \mathbf{b} = \langle 1, 0, -1 \rangle + \langle 3, -2, 1 \rangle = \langle 4, -2, 0 \rangle,$$

and

$$\mathbf{a} - 2\mathbf{b} = \langle 1, 0, -1 \rangle + 2\langle 3, -2, 1 \rangle = \langle 7, -4, 1 \rangle.$$

Question 2. Find a vector in $\mathbb{R}^2$ that has length one and is parallel to the tangent line to the curve $y = 2 \sin x$ at $(\frac{\pi}{6}, 1)$. Is your answer unique?

Solution 2. The slope of the tangent line is

$$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}} = 2 \cos x|_{x=\frac{\pi}{6}} = \sqrt{3}.$$

Therefore, the vector $\mathbf{v} = \langle 1, \sqrt{3} \rangle$ is parallel to the tangent line to the curve at the given point. From Pythagoras, we see that the length of $\mathbf{v}$ is $\sqrt{1^2 + \sqrt{3}^2} = 2$. Hence, the vector

$$\mathbf{u} = \frac{1}{2}\mathbf{v}$$

has length one.

The answer is not unique since the vector $-\mathbf{u}$ shares the same properties.