VANDERBILT UNIVERSITY
MATH 208 — ORDINARY DIFFERENTIAL EQUATIONS
PRACTICE TEST 1.

Question 1. For each equation below, identify the unknown function, classify the equation as linear or non-linear, and state its order.

(a) $y\dfrac{dy}{dx} + \dfrac{y}{x} = 0.$

(b) $x'''' + \cos t\, x' = \sin t.$

(c) $y''' = -\cos y\, y'.$

Question 2. Solve the following initial value problems.

(a) $y' = \dfrac{y-1}{x+3}$, $y(-1) = 0.$

(b) $y' = e^{-x} - 4y$, $y(0) = \dfrac{4}{3}.$

Question 3. Solve the following differential equations.
(a) $y' = \dfrac{\cos y \cos x + 2x}{\sin y \sin x + 2y}.$

(b) $y' = 2x^{-1}y + x^2 \cos x,\ \ x > 0.$

(c) $x^2 y' = y - 1.$

Question 4. Find the general solution of the given differential equation.
(a) $y'' + 8y' - 14y = 0.$

(b) $y'' + 8y' - 9y = 0.$

(c) $t^2 y'' + 5y = 0$, $t > 0.$

Question 5. Give the form of the particular solution for the given differential equations. You do not have to find the values of the constants of the particular solution.

(a) $y'' + 2y' - 3y = \cos x.$

(b) $y'' + 4y = 8\sin 2t$.

(c) $y'' - 2y' + y = e^t \cos t$.

(d) $y'' - y' - 12y = 2t^6 e^{-3t}$.

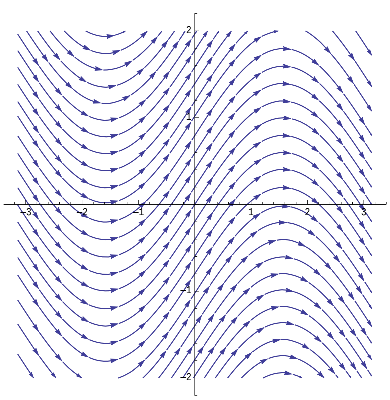
Question 6. Verify that the given functions are two linearly independent solutions of the corresponding homogeneous equation. Then, find a particular solution solving the non-homogeneous problem.

(a) $x^2 y'' - 2y = 3x^2 - 1$, $x > 0$, $y_1 = x^2$, $y_2 = x^{-1}$.

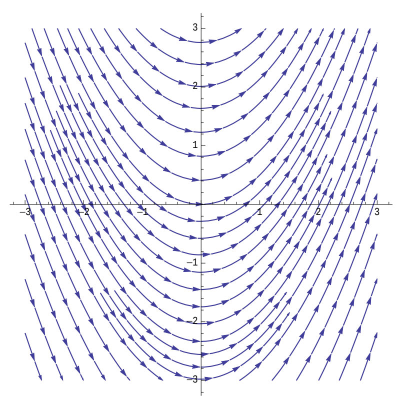
(b) $(1-x)y'' + xy' - y = \sin x$, $0 < x < 1$, $y_1 = e^x$, $y_2 = x$.

Question 7. Match the direction fields with the given differential equations.

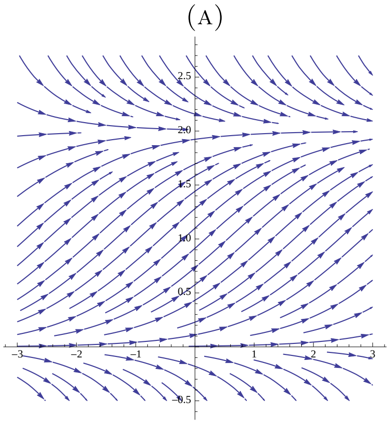
(a) $y' = -\dfrac{y}{x}$ (b) $y' = \cos x$ (c) $y' = y(1 - 0.5y)$ (d) $y' = x$



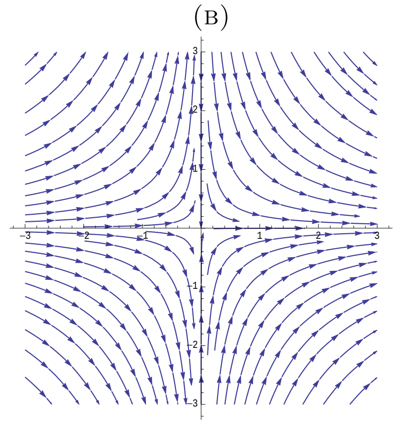
(A)



(B)



(C)



(D)

Question 8. Show that the problem

$$3y' - x^2 + xy^3 = 0, \; y(1) = 6,$$

has a unique solution defined in some neighborhood of $x = 1$.

URL: http://www.disconzi.net/Teaching/MAT208-Fall-14/MAT208-Fall-14.html