

VANDERBILT UNIVERSITY
MATH 208 — ORDINARY DIFFERENTIAL EQUATIONS
EXAMPLES OF SECTION 5.3.

Question 1. Write

$$y''' - t^2 y'' + 2t \cos(t - y') + y^3 = \sin t, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = -1,$$

as a system of first order differential equations in normal form.

Solution. Set $x_1 = y$, $x_2 = y'$, $x_3 = y''$. Then

$$\begin{aligned}x_1' &= y' = x_2, \\x_2' &= (y')' = y'' = x_3,\end{aligned}$$

and

$$\begin{aligned}x_3' &= (y'')' = y''' = t^2 y'' - 2t \cos(t - y') - y^3 + \sin t \\&= t^2 x_3 - 2t \cos(t - x_2) - x_1^3 + \sin t.\end{aligned}$$

The system is

$$\begin{cases}x_1' = x_2, \\x_2' = x_3, \\x_3' = t^2 x_3 - 2t \cos(t - x_2) - x_1^3 + \sin t, \\x_1(0) = 1, x_2(0) = 0, x_3(0) = -1.\end{cases}$$

URL: <http://www.disconzi.net/Teaching/MAT208-Fall-14/MAT208-Fall-14.html>