Question 1. Write
$$y''' - t^2 y'' + 2t \cos(t - y') + y^3 = \sin t, \ y(0) = 1, \ y'(0) = 0, y''(0) = -1,$$
as a system of first order differential equations in normal form.

Solution. Set $x_1 = y$, $x_2 = y'$, $x_3 = y''$. Then
$$x_1' = y' = x_2,$$
$$x_2' = (y')' = y'' = x_3,$$
and
$$x_3' = (y'')' = y''' = t^2 y'' - 2t \cos(t - y') - y^3 + \sin t$$
$$= t^2 x_3 - 2t \cos(t - x_2) - x_1^3 + \sin t.$$
The system is
$$\begin{cases} x_1' = x_2, \\ x_2' = x_3, \\ x_3' = t^2 x_3 - 2t \cos(t - x_2) - x_1^3 + \sin t, \\ x_1(0) = 1, x_2(0) = 0, x_3(0) = -1. \end{cases}$$

URL: http://www.disconzi.net/Teaching/MAT208-Fall-14/MAT208-Fall-14.html