VANDERBILT UNIVERSITY
MATH 208 — ORDINARY DIFFERENTIAL EQUATIONS
EXAMPLES OF SECTION 5.2.

Question 1. Find the general solution of

$$\begin{cases} x' - x + 4y = 0, \\ y' - x - y = 0. \end{cases}$$

Solutions. Expressing x in terms of y in the second equation:

$$x = y' - y. \tag{1}$$

Plugging into the first equations yields

$$x' - x + 4y = (y' - y)' - (y' - y) + 4y = y'' - 2y' + 5y = 0.$$

The characteristic equation is $\lambda^2 - 2\lambda + 5 = 0$, which has roots $1 \pm 2i$. Thus

$$y = c_1 e^t \cos 2t + c_2 e^t \sin 2t.$$

Plugging into (1) produces

$$y = 2c_2 e^t \cos 2t - 2c_1 e^t \sin 2t.$$