

VANDERBILT UNIVERSITY
MATH 198 —METHODS OF ORDINARY DIFFERENTIAL EQUATIONS
EXAMPLES OF SECTION 7.8.

Question. Solve

$$\begin{cases} y'' + y = \delta(t - \pi) - \delta(t - 2\pi), \\ y(0) = 0, y'(0) = 1. \end{cases}$$

Solution. Applying the Laplace transform,

$$\begin{aligned} \mathcal{L}\{y'' + y\} &= \mathcal{L}\{\delta(t - \pi) - \delta(t - 2\pi)\} \\ s^2 Y(s) - sy(0) - y'(0) + Y(s) &= e^{-\pi s} - e^{-2\pi s} \\ s^2(Y(s) + 1) &= 1 + e^{-\pi s} - e^{-2\pi s}. \end{aligned}$$

Thus,

$$Y(s) = \frac{1}{s^2 + 1} + \frac{e^{-\pi s}}{s^2 + 1} - \frac{e^{-2\pi s}}{s^2 + 1}.$$

Using

$$\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t - a)u(t - a),$$

where $F(s) = \mathcal{L}\{f(t)\}$, we obtain, with $F(s) = \frac{1}{s^2 + 1}$,

$$y(t) = \sin t + \sin(t - \pi)u(t - \pi) - \sin(t - 2\pi)u(t - 2\pi).$$

This can be simplified to

$$y(t) = \sin t(1 - u(t - \pi) - u(t - 2\pi)).$$

URL: <http://www.disconzi.net/Teaching/MAT198-Spring-14/MAT198-Spring-14.html>