# VANDERBILT UNIVERSITY
# MATH 196 — DIFFERENTIAL EQUATIONS WITH LINEAR ALGEBRA
# EXAMPLES OF SECTIONS 3.1.

**Question 1.** Solve the linear system:

$$\begin{cases} x + 3y + 2z = 2 \\ 2x + 7y + 7z = -1 \\ 2x + 5y + 2z = 7 \end{cases}$$

**Question 2.** Identify a matrix that describes the previous system.

**Question 3.** Consider the differential equation $y'' - 10y + 21y = 0$. Verify that $y = Ae^{3x} + Be^{7x}$ is a solution, with $A$ and $B$ arbitrary constants. If in addition $y(0) = 15$, $y'(0) = 13$, write a system for $A$ and $B$.

**SOLUTIONS.**

**1.** Subtract twice the first equation from the second one and replace the second equation by the result to get

$$\begin{cases} x + 3y + 2z = 2 \\ y + 3z = -5 \\ 2x + 5y + 2z = 7 \end{cases}$$

Subtract twice the first equation from the third one and replace the third equation by the result to get

$$\begin{cases} x + 3y + 2z = 2 \\ y + 3z = -5 \\ -y + -2z = 3 \end{cases}$$

Adding the last two equations

$$\begin{cases} x + 3y + 2z = 2 \\ y + 3z = -5 \\ z = -2 \end{cases}$$

Multiply the third equation by $-3$, add to the second one and replace the second equation with the result to get

$$\begin{cases} x + 3y + 2z = 2 \\ y = 1 \\ z = -2 \end{cases}$$

Multiply the third equation by $-2$, add to the first one to obtain

$$\begin{cases} x + 3y = 6 \\ y = 1 \\ z = -2 \end{cases}$$

Multiply the second equation by $-3$ and add to the first one to finally obtain

$$\begin{cases} x = 3 \\ y = 1 \\ z = -2 \end{cases}$$

So the solution of the system is $x = 3$, $y = 1$, $z = -2$.

**2.** From the original system we read off

$$\begin{bmatrix} 1 & 3 & 2 & 2 \\ 2 & 7 & 7 & -1 \\ 2 & 5 & 2 & 7 \end{bmatrix}$$

**3.** We have

$$y = Ae^{3x} + Be^{7x},$$

$$y' = 3Ae^{3x} + 7Be^{7x},$$

$$y'' = 9Ae^{3x} + 49Be^{7x}.$$

Then

$$\begin{aligned} y'' - 10y + 21y &= 9Ae^{3x} + 49Be^{7x} - 10(3Ae^{3x} + 7Be^{7x}) + 21(Ae^{3x} + Be^{7x}) \\ &= (9A - 30A + 21A)e^{3x} + (49B - 70B + 21B)e^{7x} \\ &= 0. \end{aligned}$$

Plugging zero into the expression for $y$ and using $y(0) = 15$ we find $A + B = 15$, and plugging zero into the expression for $y'$ and using $y'(0) = 13$ we obtain $3A + 7B = 13$, so

$$\begin{cases} A + B = 15 \\ 3A + 7B = 13 \end{cases}$$