VANDERBILT UNIVERSITY
MATH 196 — DIFFERENTIAL EQUATIONS WITH LINEAR ALGEBRA
EXTRA CREDIT ASSIGNMENT.

In the problems 1 to 3 below, V denotes the set of all continuous functions defined on the interval $[0, 1]$.

Question 1. Show that V is a vector space.

Question 2. For any two $f, g \in V$, let $\langle f, g \rangle$ be given by

$$\langle f, g \rangle = \int_0^1 f(x)g(x)\, dx.$$

Show that $\langle \, , \, \rangle$ is an inner product on V, i.e. it satisfies all the properties on page 270 of the textbook.

Question 3. Using the inner product above, show that the functions $\cos(m\pi x)$ and $\sin(n\pi x)$ are linearly independent for any integers m and n. What can you conclude about the dimension of V?

Question 4. Read section 1.3 of the textbook, paying special attention to examples 1 and 2. Do problems 1, 3, 21, and 23 of section 1.3.