

## 11.7

12.) Since  $0 < \frac{1}{k\sqrt{k^{2+1}}} < \frac{1}{k\sqrt{k^2}} = \frac{1}{k^2}$ , and

$$\sum_{k=1}^{\infty} \frac{1}{k^2} \text{ converges, } \sum_{k=1}^{\infty} \frac{1}{k\sqrt{k+1}} \text{ converges.}$$

18.) Since  $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}-1} = 0$  and  $\frac{1}{\sqrt{n+1}-1} < \frac{1}{\sqrt{n}-1}$ ,

by the Alternating Series Test,  $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}-1}$   
converges.

22.) Since  $\frac{1}{3} \leq \frac{1}{2+\sin k}$ ,  $\lim_{k \rightarrow \infty} \frac{1}{2+\sin k}$  (if it exists)

must be greater than or equal to  $\frac{1}{3}$ . Thus, by

the Test for Divergence,  $\sum_{k=1}^{\infty} \frac{1}{2+\sin k}$  diverges.

16.) Since  $\lim_{n \rightarrow \infty} \frac{\left(\frac{n^2+1}{n^3+1}\right)}{\left(\frac{1}{n}\right)} = \lim_{n \rightarrow \infty} \frac{n^3+n}{n^3+1} = 1$ , and

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ diverges, } \sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1} \text{ diverges.}$$

$$28.) \text{ Since } \int_1^{\infty} \frac{e^{1/x}}{x^2} dx = \lim_{t \rightarrow \infty} \int_1^t \frac{e^{1/x}}{x^2} dx = \lim_{t \rightarrow \infty} \int_1^{1/t} -e^v dv$$

$$\text{Let } v = \frac{1}{x}.$$

$$\text{Then, } dv = -\frac{1}{x^2} dx.$$

$$= \lim_{t \rightarrow \infty} e^v \Big|_1^{1/t} = \lim_{t \rightarrow \infty} (e - e^{1/t})$$

$$= e - 1, \quad \sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2} \text{ converges.}$$