MAT 155B - FALL 12 - EXAMPLES SECTION 8.2

Question. Write an integral representing the area of the surface obtained by rotating the curve $y = e^{2x} \sin x$, $-1 \leq x \leq 2$, about $x = 4$.

Solution. Put $u = x - 4$, so the curve becomes

$$y = e^{2u+8} \sin(u+4), \ -5 \leq u \leq -2,$$

rotated about the u-axis. Compute

$$\frac{dy}{du} = 2e^{2u+8} \sin(u+4) + e^{2u+8} \cos(u+4).$$

The formula for the area is

$$S = \int_a^b 2\pi u \sqrt{1 + \left(\frac{dy}{du}\right)^2}\, du,$$

so

$$\begin{aligned} S &= \int_{-5}^{-2} 2\pi u \sqrt{1 + \left(\frac{dy}{du}\right)^2}\, du \\ &= \int_{-5}^{-2} 2\pi u \sqrt{1 + (2e^{2u+8} \sin(u+4) + e^{2u+8} \cos(u+4))^2}\, du. \end{aligned}$$