MAT 155B - FALL 12 - EXAMPLES SECTION 11.6

Question. Determine whether the series converges.

(a) $\displaystyle\sum_{n=3}^{\infty} \frac{(2n)!}{4^n}$.

(b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{10^n}{n!}$.

Solution a. Let us use the ration test:

$$\frac{a_{n+1}}{a_n} = \frac{\frac{(2(n+1))!}{4^{n+1}}}{\frac{(2n)!}{4^n}}$$

$$= \frac{(2(n+1))!}{4^{n+1}} \frac{4^n}{(2n)!}$$

$$= \frac{(2n+2)(2n+1)}{4} \to \infty \text{ as } n \to \infty,$$

so the series diverges by the ratio test.

Solution b. Let us use the ration test:

$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{\frac{10^{n+1}}{(n+1)!}}{\frac{10^n}{n!}}$$

$$= \frac{10^{n+1}}{(n+1)!} \frac{n!}{10^n}$$

$$= \frac{10}{n+1} \to 0 \text{ as } n \to \infty.$$

Hence the series is absolutely convergent by the ratio test, and therefore convergent.