MAT 155B - FALL 12 - EXAMPLES OF SECTION 10.4

Question 1. Find the area of the region enclosed by one loop of the curve

$$r = 2\sin 5\theta.$$

Question 2. Find the exact length of the curve

$$r = 5^{\theta},\ 0 \leq \theta \leq 2\pi.$$

Solutions.

1. Since at $\theta = 0$ we have $r = 0$, the first loop of the curve will be completed when $r = 0$ again, thus

$$r = 2\sin 5\theta = 0 \Rightarrow 5\theta = n\pi,\ n = 0, 1, 2, \dots$$

Because $\theta = 0$ is the starting point, the first loop closes when $n = 1$, i.e., $\theta = \frac{\pi}{5}$. Therefore

$$A = \int_0^{\frac{\pi}{5}} \frac{1}{2}(2\sin 5\theta)^2\, d\theta = \frac{1}{2}\int_0^{\frac{\pi}{5}} 4\sin^2 5\theta\, d\theta = 2\int_0^{\frac{\pi}{5}} \frac{1}{2}(1 - \cos 10\theta)\, d\theta = \frac{\pi}{5}.$$

2. Simply compute

$$\begin{aligned}
L &= \int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \\
&= \int_0^{2\pi} \sqrt{(5^{\theta})^2 + (5^{\theta}\ln 5)^2}\, d\theta \\
&= \int_0^{2\pi} \sqrt{5^{2\theta}(1 + (\ln 5)^2)}\, d\theta \\
&= \sqrt{1 + (\ln 5)^2} \int_0^{2\pi} \sqrt{5^{2\theta}}\, d\theta \\
&= \sqrt{1 + (\ln 5)^2} \int_0^{2\pi} 5^{\theta}\, d\theta = \sqrt{1 + (\ln 5)^2}\, \frac{5^{\theta}}{\ln 5}\bigg|_0^{2\pi} \\
&= \frac{\sqrt{1 + (\ln 5)^2}}{\ln 5}(5^{2\pi} - 1).
\end{aligned}$$

URL: http://www.disconzi.net/Teaching/MAT155B-Fall-12/MAT155B-Fall-12.html