MAT 155B - FALL 12 - EXAMPLES OF SECTION 10.3

Question 1. Identify the curve given in polar coordinates by

$$r = 4\sec\theta.$$

Question 2. Find the slope of the tangent line to the given polar curve at the point specified by the value of θ:

$$\text{(a)}\quad r = 2 - \sin\theta,\ \theta = \frac{\pi}{3}, \qquad\qquad \text{(b)}\quad r = \cos\frac{\theta}{3},\ \theta = \pi.$$

Solutions.

1. Write

$$r = 4\sec\theta \Rightarrow r = \frac{4}{\cos\theta} \Rightarrow 4 = r\cos\theta.$$

Since for any θ and any r, we have

$$x = r\cos\theta,$$

we conclude that the equation $r = 4\sec\theta$ represents the line $x = 4$.

2. First, let us find a formula for the slope at a point (r, θ). By the chain rule we have

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}.$$

(compare with formula (1) on page 669, why are they so similar?). Since $y = r\sin\theta$, we have

$$\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta,$$

and similarly, since $x = r\cos\theta$,

$$\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta.$$

Therefore

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}. \tag{1}$$

2a. From $r = 2 - \sin\theta$, compute

$$\frac{dr}{d\theta} = -\cos\theta,$$

and plug in (1) to find

$$\frac{dy}{dx} = \frac{-\cos\theta\sin\theta + (2-\sin\theta)\cos\theta}{-\cos\theta\cos\theta - (2-\sin\theta)\sin\theta} = \frac{2\cos\theta - 2\cos\theta\sin\theta}{-2\sin\theta + \sin^2\theta - \cos^2\theta}.$$

Plugging $\theta = \frac{\pi}{3}$ we find

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{2-\sqrt{3}}{1-2\sqrt{3}}.$$

2b. From $r = \cos\frac{\theta}{3}$, compute

$$\frac{dr}{d\theta} = -\frac{1}{3}\sin\frac{\theta}{3}.$$

Therefore, using (1),

$$\frac{dy}{dx} = \frac{-\frac{1}{3}\sin\frac{\theta}{3}\sin\theta + \cos\frac{\theta}{3}\cos\theta}{-\frac{1}{3}\sin\frac{\theta}{3}\cos\theta - \cos\frac{\theta}{3}\sin\theta}.$$

Plugging in $\theta = \pi$ gives

$$\left.\frac{dy}{dx}\right|_{\theta=\pi} = -\sqrt{3}.$$

URL: `http://www.disconzi.net/Teaching/MAT155B-Fall-12/MAT155B-Fall-12.html`