MATH 155B, Quiz 8
November 15, 2012
Name:

KEY

You have 10 minutes to complete this quiz. The use of calculators is not permitted.
**Show all work if you want full credit for your solutions. Zero credit will
be given for answers with zero work shown, even if the answer is correct.**
Good luck!

(1) Find the radius of convergence and interval of convergence of the series
$$\sum_{n=1}^{\infty} \frac{10^n x^n}{n^3}.$$

$$\lim_{n \to \infty} \left| \frac{\left(\frac{10^{n+1} x^{n+1}}{(n+1)^3}\right)}{\left(\frac{10^n x^n}{n^3}\right)} \right| = \lim_{n \to \infty} \left| \frac{10x}{\left(\frac{n+1}{n}\right)^3} \right| = 10|x| < 1 \quad \text{if} \quad |x| < \frac{1}{10}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{1}{n^3} \quad \text{converge}$$

radius of convergence is $\frac{1}{10}$

interval of convergence is

$$\left[\frac{-1}{10}, \frac{1}{10} \right]$$

(2) Find the Maclaurin series for $f(x) = 3^x$, using the definition of a Maclaurin
series. Also, find the associated radius of convergence.

$f(x) = 3^x$

$f'(x) = 3^x \ln 3$

$f''(x) = 3^x (\ln 3)^2$

$\vdots$

$$3^x = \sum_{n=0}^{\infty} \frac{(\ln 3)^n x^n}{n!} \quad , \quad \text{radius of convergence is } \infty$$

$$\lim_{n \to \infty} \left| \frac{\left(\frac{(\ln 3)^{n+1} x^{n+1}}{(n+1)!}\right)}{\left(\frac{(\ln 3)^n x^n}{n!}\right)} \right| = \lim_{n \to \infty} \left| \frac{\ln 3 \cdot x}{n+1} \right| = 0 < 1$$

I have neither given nor received aid on this _____

1