MATH 155B, Quiz 7
November 8, 2012
Name: KEY

You have 20 minutes to complete this quiz. The use of calculators is not permitted. **Show all work if you want full credit for your solutions. Zero credit will be given for answers with zero work shown, even if the answer is correct.** Good luck!

(1) Use the Ratio Test to determine whether the series $\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$ is convergent or divergent.

$$\lim_{n\to\infty} \frac{\left(\frac{(n+1)!}{e^{(n+1)^2}}\right)}{\left(\frac{n!}{e^{n^2}}\right)} = \lim_{n\to\infty} \frac{n+1}{e^{(n^2+2n+1)-n^2}} = \lim_{n\to\infty} \frac{n+1}{e^{2n+1}} = 0 < 1$$

By the Ratio Test, the series converges.

(2) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{\sqrt{n}}$ is absolutely convergent, conditionally convergent, or divergent.

- $$\lim_{n\to\infty} \frac{\ln n}{\sqrt{n}} = 0, \quad \frac{d}{dx}\left(\frac{\ln x}{\sqrt{x}}\right) = \frac{\frac{\sqrt{x}}{x} - \frac{\ln x}{2\sqrt{x}}}{x} = \frac{2-\ln x}{2x^{3/2}} < 0$$
when $x > e^2$

By the Alternating Series Test, the series converges.

I have neither given nor received aid on this ____________________

- $\frac{\ln n}{\sqrt{n}} > \frac{1}{\sqrt{n}}$, for $n > 2$, so the series conditionally converges