

MATH 155B, Quiz 4

September 27, 2012

Name:

KEY

You have 15 minutes to complete this quiz. The use of calculators is not permitted. Show all work if you want full credit for your solutions. Zero credit will be given for answers with zero work shown, even if the answer is correct. Good luck!

(1) Let N be any positive number. Find a $t > 1$ such that $\int_1^t \frac{(\ln x)^2}{x} dx = N$.

$$N = \int_1^t \frac{(\ln x)^2}{x} dx = \int_0^{\ln t} u^2 du = \left. \frac{1}{3} u^3 \right|_0^{\ln t} = \frac{1}{3} (\ln t)^3, \text{ so } \boxed{t = e^{\sqrt[3]{3N}}} \quad \left| \begin{array}{l} \text{Let } u = \ln x. \\ \text{Then, } du = \frac{dx}{x}. \end{array} \right.$$

(2) Evaluate $\int \frac{2x^3 + x^2 + 7x + 3}{x^4 + 6x^2 + 9} dx$.

$$\frac{2x^3 + x^2 + 7x + 3}{(x^2 + 3)^2} = \frac{Ax + B}{x^2 + 3} + \frac{Cx + D}{(x^2 + 3)^2}, \quad 2x^3 + x^2 + 7x + 3 = (Ax + B)(x^2 + 3) + (Cx + D)$$

$$= Ax^3 + Bx^2 + (3A + C)x + (3B + D)$$

$$\int \frac{2x^3 + x^2 + 7x + 3}{x^4 + 6x^2 + 9} dx = \int \frac{2x + 1}{x^2 + 3} dx + \int \frac{x}{(x^2 + 3)^2} dx$$

$$= \int \frac{2x}{x^2 + 3} dx + \int \frac{dx}{x^2 + 3} + \int \frac{x}{(x^2 + 3)^2} dx$$

$$\begin{array}{l} A = 2 \\ B = 1 \\ 3A + C = 7, \quad C = 1 \\ 3B + D = 3, \quad D = 0 \end{array}$$

Let $v = x^2 + 3$.
Then, $dv = 2x dx$

I have neither given nor received aid on this _____

$$= \int \frac{dv}{v} + \frac{1}{2} \int \frac{dv}{v^2} + \int \frac{dx}{x^2 + 3} = \ln|v| - \frac{1}{2v} + \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

$$\boxed{= \ln(x^2 + 3) - \frac{1}{2(x^2 + 3)} + \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C}$$