

MATH 155B, Quiz 2

September 6, 2012

Name:

KEY

You have 10 minutes to complete this quiz. The use of calculators is not permitted. Show all work if you want full credit for your solutions. Zero credit will be given for answers with zero work shown, even if the answer is correct. Good luck!

(1) A sample of tritium-3 decayed to 92% of its original amount after 4 years.

(a) What is the half-life of tritium-3?

$$0.92 = e^{4k}, \quad \ln(0.92) = 4k, \quad k = \frac{\ln(0.92)}{4}$$

$$t_{1/2} = \frac{-\ln 2}{k} = \frac{-4 \ln 2}{\ln(0.92)}$$

(b) How long would it take the sample to decay to 10% of its original amount?

$$0.1 = e^{\left(\frac{\ln(0.92)}{4}\right)t}, \quad \ln(0.1) = \frac{\ln(0.92)}{4}t,$$

$$t = \frac{4 \ln(0.1)}{\ln(0.92)}$$

(2) Evaluate $\int_0^{\sqrt{\frac{1}{2}}} \frac{x+x^3}{\sqrt{1-x^4}} dx$.

$$= \int_0^{\sqrt{\frac{1}{2}}} \frac{x}{\sqrt{1-x^4}} dx + \int_0^{\sqrt{\frac{1}{2}}} \frac{x^3}{\sqrt{1-x^4}} dx$$

$$= \frac{1}{2} \int_0^{\sqrt{\frac{1}{2}}} \frac{dv}{\sqrt{1-v^2}} + \left(-\frac{1}{4}\right) \int_1^{\frac{1}{2}} \frac{1}{\sqrt{w}}$$

$$= \frac{1}{2} \sin^{-1}(v) \Big|_0^{\sqrt{\frac{1}{2}}} + \left(-\frac{1}{2} \sqrt{w}\right) \Big|_1^{\frac{1}{2}}$$

$$v = x^2 \\ dv = 2x dx$$

$$w = 1-x^4 \\ dw = -4x^3 dx$$

I have neither given nor received aid on this _____

$$= \frac{1}{2} \sin^{-1}\left(\sqrt{\frac{1}{2}}\right) - \frac{1}{2} \left(\sqrt{\frac{1}{2}} - 1\right)$$

$$= \frac{\pi}{8} - \frac{\sqrt{2}}{4} + \frac{1}{2}$$