VANDERBILT UNIVERSITY
MAT 155B, FALL 12 — PRACTICE TEST 4

Important: The goal of the practice test is to give you an idea of the sort of questions which will be asked in the exam (e.g., finding a Taylor series, computing the sum of a series, etc), and which sections of the textbook will have more emphasis. The practice test *does not* indicate what *type* of series will appear in the test. For example, because in problem 3 below there is no question with $\ln x$, it does not mean that you should not study Taylor and Maclaurin series involving $\ln x$ (in fact, you should know/memorize all the series on table 1, page 786, of the textbook).

Question 1. Find the radius and interval of convergence of the series.

(a). $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^2 5^n}$

(b). $\displaystyle\sum_{n=1}^{\infty} n\frac{(x+1)^n}{4^n}$

(c). $\displaystyle\sum_{n=1}^{\infty} \frac{n!x^n}{1\cdot 3\cdot \cdots \cdot (2n-1)}$

Question 2. Find the radius of convergence of

$$\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2} x^n.$$

Question 3. Find the Maclaurin series for $f(x)$ and its radius of convergence.

(a). $f(x) = \dfrac{x^2}{1+x}$

(b). $f(x) = 10^x$

(c). $f(x) = \cosh x$

(d). $f(x) = \sin^2 x$

(e). $f(x) = xe^{-x}$

Question 4. Find the Taylor series for $f(x)$ centered at the given value of a.

(a). $f(x) = \dfrac{1}{x},\ a = -3$

(b). $f(x) = \sqrt{x},\ a = 16$

(c). $f(x) = \cos x,\ a = \pi$

Question 5. Evaluate the integral as an infinite series.

(a). $\displaystyle\int \arctan(x^2)\,dx$

(b). $\displaystyle\int \frac{e^x - 1}{x}\,dx$

(c). $\displaystyle\int x\cos(x^3)\,dx$

Question 6. Use series to approximate the definite integral to within the indicated accuracy.

(a). $\displaystyle\int_0^1 \sin(x^4)\,dx,$ four decimal places

(b). $\displaystyle\int_0^1 e^{-x^2}\,dx,$ three decimal places

Question 7. Find the sum of the series.

(a). $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^n}{3^{2n}(2n)!}$

(b). $\displaystyle\frac{1}{1\cdot 2} - \frac{1}{3\cdot 2^3} + \frac{1}{5\cdot 2^5} - \frac{1}{7\cdot 2^7} + \cdots$

Question 8. Find the points on the given curve where the tangent line is either horizontal or vertical.

(a). $r = 3\cos\theta$

(b). $r = 1 + \cos\theta$

Question 9. Write an integral representing the area enclosed by one loop of the given curve (do not evaluate the integral).

(a). $r = 4\cos 3\theta$

(b). $r^2 = \sin 2\theta$

Question 10. Write an integral representing the length of the given curve (do not evaluate the integral).

(a). $\displaystyle r = \frac{1}{\theta},\ \pi \le \theta \le 2\pi$

(b). $\displaystyle r = \sin^3\frac{\theta}{3},\ 0 \le \theta \le \pi$