MAT 155B - FALL 12 - ASSIGNMENT 1
SOLUTIONS

1. Prove the following identity:

$$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y.$$

2. Compute the derivative of the given functions:

(a) $y = \sinh x \ln \tanh x$.

(b) $y = \sinh^{-1}(\arccos x)$.

Solutions.

1. Simply compute:

$$\begin{aligned}\cosh x \cosh y + \sinh x \sinh y &= \frac{(e^x + e^{-x})(e^y + e^{-y})}{4} + \frac{(e^x - e^{-x})(e^y - e^{-y})}{4} \\ &= \frac{e^{x+y} + e^{-(x+y)} + e^{x-y} + e^{-x+y} + e^{x+y} + e^{-(x+y)} - e^{x-y} - e^{-x+y}}{4} \\ &= \frac{e^{x+y} + e^{-(x+y)}}{2} = \cosh(x+y).\end{aligned}$$

2a. Use the product rule, the chain rule and $(\sinh x)' = \cosh x$, $(\tanh x)' = \operatorname{sech}^2 x$ to find

$$y' = \cosh x \ln \tanh x + \sinh x \frac{\operatorname{sech}^2 x}{\tanh x} = \cosh x \ln \tanh x + \operatorname{sech} x.$$

2a. Use the the chain rule and $(\sinh^{-1} x)' = \frac{1}{1+x^2}$, and $(\cos^{-1} x)' = -\frac{1}{1-x^2}$ to find

$$y' = \frac{1}{\sqrt{1 + (\cos^{-1} x)^2}} (\cos^{-1} x)' = -\frac{1}{\sqrt{1 + (\cos^{-1} x)^2}\sqrt{1 - x^2}}.$$

URL: `http://www.disconzi.net/Teaching/MAT155B-Fall-12/MAT155B-Fall-12.html`