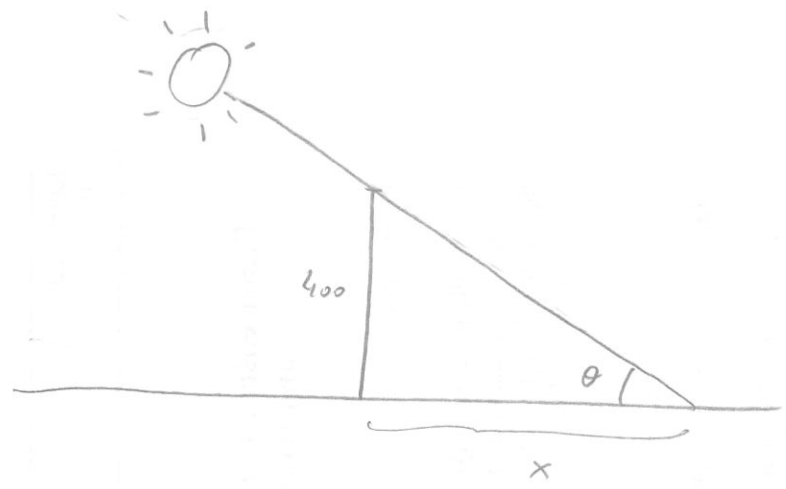
5



$$\frac{d\theta}{dt} = -0.25 \text{ rad/h}$$

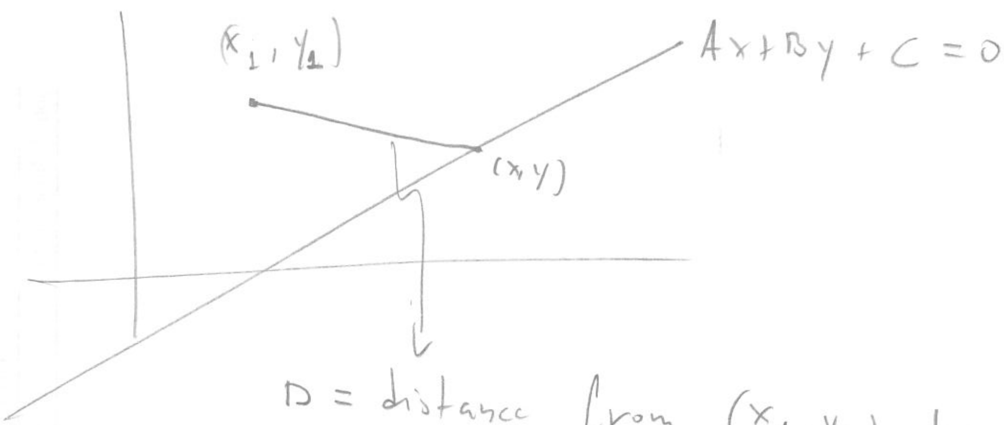
$$\tan\theta = \frac{400}{x} \Rightarrow x = \frac{400}{\tan\theta} = 400\cot\theta$$

$$\frac{dx}{dt} = -400\csc^2\theta \underbrace{\frac{d\theta}{dt}}_{=-1/4} = +\frac{100}{\sin^2\theta}$$

$$\left.\frac{dx}{dt}\right|_{\theta=\frac{\pi}{6}} = \frac{100}{(1/2)^2} = 400 \text{ ft/h}$$

question 5

(6)



D = distance from (x_1, y_1) to a point (x, y) on the line.

The problem is trivial if $B = 0$, so assume $B \neq 0$.

Write

$$D^2 = (x - x_1)^2 + (y - y_1)^2$$

From $Ax + By + C = 0 \Rightarrow y = -\frac{A}{B}x - \frac{C}{B}$,

thus

$$D^2 = (x - x_1)^2 + \left(-\frac{A}{B}x - \frac{C}{B} - y_1\right)^2$$

$$= (x - x_1)^2 + \left(\frac{A}{B}x + \frac{C}{B} + y_1\right)^2$$

Let $f(x) = D^2$

question 6, 1

$$f'(x) = 2(x - x_1) + 2\left(\frac{A}{B}x + \frac{C}{B} + y_1\right)\cdot\frac{A}{B}$$

$$= 2x\left(1 + \frac{A^3}{B^2}\right) + \frac{2A}{B}\left(\frac{C}{B} + y_1\right) - 2x_1$$

Set $f'(x) = 0$,

$$x = \frac{2x_1 - \frac{2A}{B}\left(\frac{C}{B} + y_1\right)}{2\left(1 + \frac{A^2}{B^2}\right)} = \frac{\frac{B^2 x_1 - AC - Ay_1 B}{B^2}}{\frac{A^2 + B^2}{B^2}}$$

$$= \frac{B^2 x_1 - ABy_1 - AC}{A^2 + B^2}.$$

Since $f''(x) = 2\left(1 + \frac{A^2}{B^2}\right) > 0$, we see that is a minimum.

Plugging back

$$D^2 = \left(\frac{B^2 x_1 - AB y_1 - AC}{A^2+B^2} - x_1^2\right)^2 + \left(\frac{A}{B}\,\frac{B^2 x_1 - AB y_1 - AC}{A^2+B^2} + \frac{C}{B} + y_1\right)^2$$

$$= \left(\frac{-A^2 x_1 - AB y_1 - AC}{A^2+B^2}\right)^2 + \left(\frac{AB x_1 + CB + B^2 y_1}{A^2+B^2}\right)^2$$

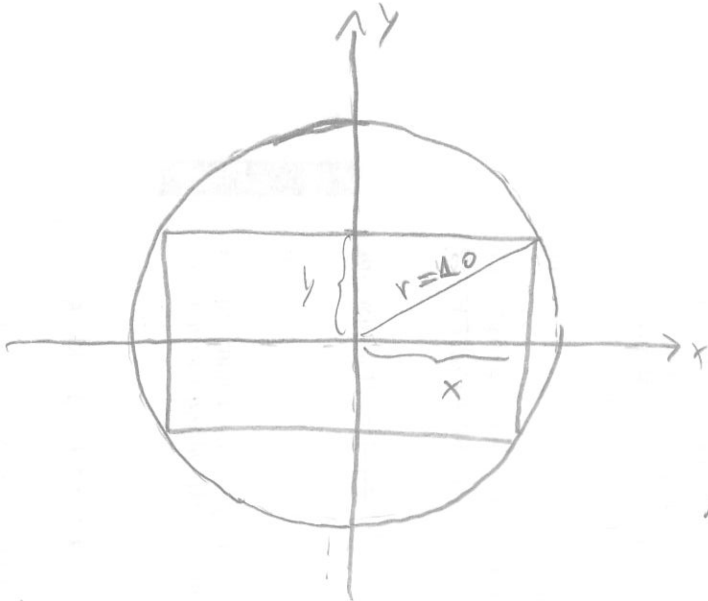
$$= \frac{A^2}{(A^2+B^2)^2}\left(A x_1 + B y_1 + C\right)^2 + \frac{B^2}{(A^2+B^2)^2}\left(A x_1 + C + B y_1\right)^2$$

$$= \frac{1}{A^2+B^2}\left(A x_1 + B y_1 + C\right)^2$$

$$\Rightarrow \quad D = \sqrt{D^2} = \frac{|A x_1 + B y_1 + C|}{\sqrt{A^2+B^2}}$$

question 6,3

(7A)



$x^2 + y^2 = 100$

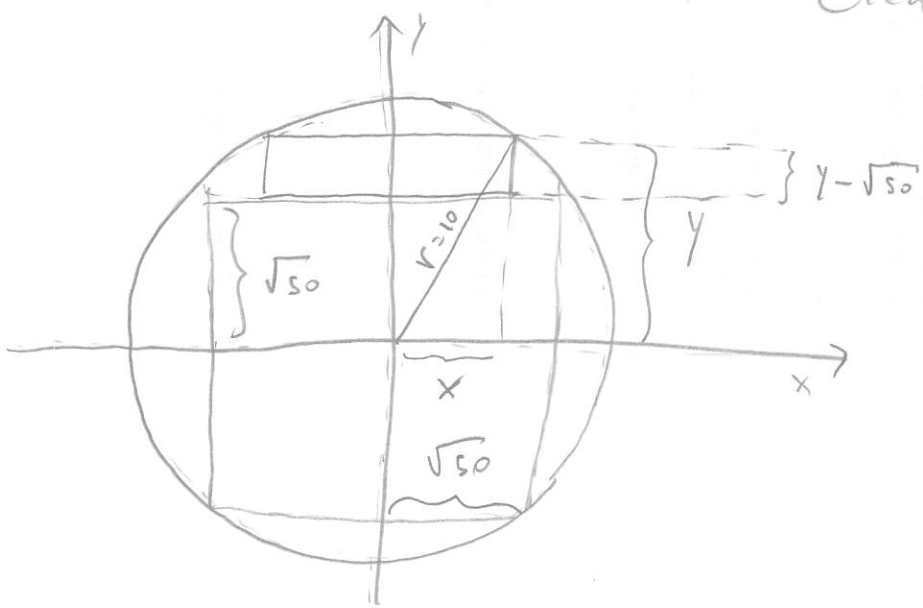
$y = \sqrt{100 - x^2}$ (we can take $y \geq 0$)

$A = 4xy = 4x\sqrt{100 - x^2}$, $0 \leq x \leq 10$

$$A' = 4\sqrt{100 - x^2} + 4x \frac{(-2x)}{2\sqrt{100 - x^2}} = \frac{4}{\sqrt{100 - x^2}}\left(100 - 2x^2\right)$$

$A' = 0 \Rightarrow x = \sqrt{50}$ Clearly a maximum, and $y = \sqrt{50}$ too

(7B)



$x^2 + y^2 = 10$

$A = 2x \cdot (y - \sqrt{50}) = 2x\left(\sqrt{100 - x^2} - \sqrt{50}\right)$

$$A' = 2\left(\sqrt{100 - x^2} - \sqrt{50}\right) + $$
$$+ 2x \frac{-2x}{2\sqrt{100 - x^2}}$$

question 7, 1

$$A' = 2\sqrt{100 - x^2} - \frac{2x^2}{\sqrt{100 - x^2}} - 2\sqrt{50} = 0$$

multiply by $\frac{\sqrt{100 - x^2}}{2}$

$$\Rightarrow \quad 100 - x^2 - x^2 - \sqrt{50}\sqrt{100 - x^2} = 0$$

$$-2x^2 + 100 = \sqrt{50}\sqrt{100 - x^2}$$, square both sides

$$4x^4 + 10000 - 400x^2 = 50(100 - x^2)$$

$$4x^4 - 350x^2 + 5000 = 0$$

Use quadradict formula for x^2:

$$\begin{array}{r} 35 \\ 35 \\ \hline 175 \\ 105 \\ \hline 1225 \end{array}$$

$$x^2 = \frac{-(-350) \pm \sqrt{(-350)^2 - 4 \cdot 4 \cdot 5000}}{2 \cdot 4}$$

$$= \left(350 \pm \sqrt{122500 - 80000}\right) / 8 = \frac{350 \pm \sqrt{42500}}{8}$$

$\sqrt{42500} = \sqrt{425 \cdot 100} = \sqrt{425} \cdot 10$, but

$\sqrt{400} < \sqrt{425} < \sqrt{441} \Rightarrow 20 < \sqrt{425} < 21$, take $\sqrt{425} \simeq 20.5$

Question 7,2

$$x^2 = \frac{350 \pm 205}{8} \quad \frac{555}{8} \quad \frac{145}{8}$$

$$555 \div 8 = 69.37 \simeq 69.4$$

$$145 \div 8 = 18.1 \simeq 18.1$$

$x^2 = 69.4, \quad x^2 = 18$

$\sqrt{64} < \sqrt{69.4} < \sqrt{81} \Rightarrow 8 < \sqrt{69.4} < 9$

Take $x \simeq 8.5$

$\sqrt{16} < \sqrt{18} < \sqrt{25} \Rightarrow 4 < \sqrt{18} < 5$ (only the positive roots matter)

Take $x \simeq 4.5$

From the picture we see that we need $x < \sqrt{50}$. But $\sqrt{50} < \sqrt{64} = 8$, so $8.5 > 8 > \sqrt{50}$, thus $x = 8.5$ doesn't work.

Then $x = 4.5$. Thus the dimensions are

$$2x \text{ by } y = 9 \text{ by } \sqrt{100 - (4.5)^2} - \sqrt{50}$$

Question 7.3

7C

Using the picture of 7A,

$$S = k\, 2x \cdot (2y)^2 = 8kxy^2 = 8kx(100 - x^2) = 800kx - 8kx^3$$

k: proportionality constant

$$S' = 800k - 24kx^2 = 0 \Rightarrow x^2 = \frac{800}{24} = \frac{100}{3} \Rightarrow x = \frac{10}{\sqrt{3}}$$

$(x > 0)$

$S'' = -24k \cdot 2 < 0$ since we can assume $k > 0$, so

$x = \frac{10}{\sqrt{3}}$ is a max.

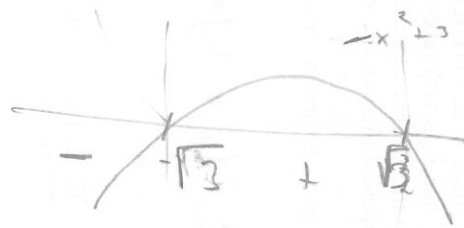
(9A)

$f(x) = \dfrac{x^2-1}{x^3}$ $\quad x=0$ is a V.A , $\lim\limits_{x\to 0^+} f(x) = -\infty$, $\lim\limits_{x\to 0^-} f(x) = +\infty$

$f'(x) = \dfrac{x^3(2x-0) - (x^2-1)\cdot 3x^2}{x^6} = \dfrac{2x^4 - 3x^4 + 3x^2}{x^6} = \dfrac{-x^4 + 3x^2}{x^6}$

$f'(x) = \dfrac{-x^2+3}{x^4}$

$-x^2+3 = 0 \Rightarrow x = \pm\sqrt{3}$

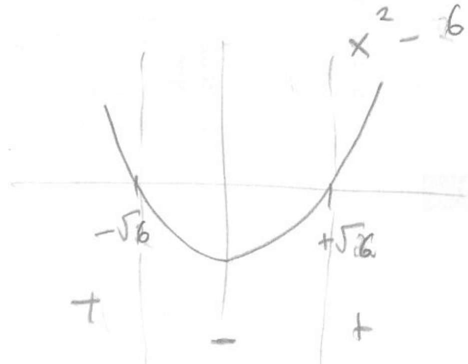


the sign of $f'(x)$ is the same because $x^4 > 0$

$f''(x) = \dfrac{x^4(-2x+0) - (-x^2+3)\,4x^3}{x^8} = \dfrac{-2x^5 + 4x^5 - 12x^3}{x^8} = \dfrac{2x^5 - 12x^3}{x^8}$

$f''(x) = \dfrac{2}{x^5}(x^2-6)$

$x^2-6=0 \Rightarrow x = \pm\sqrt{6}$

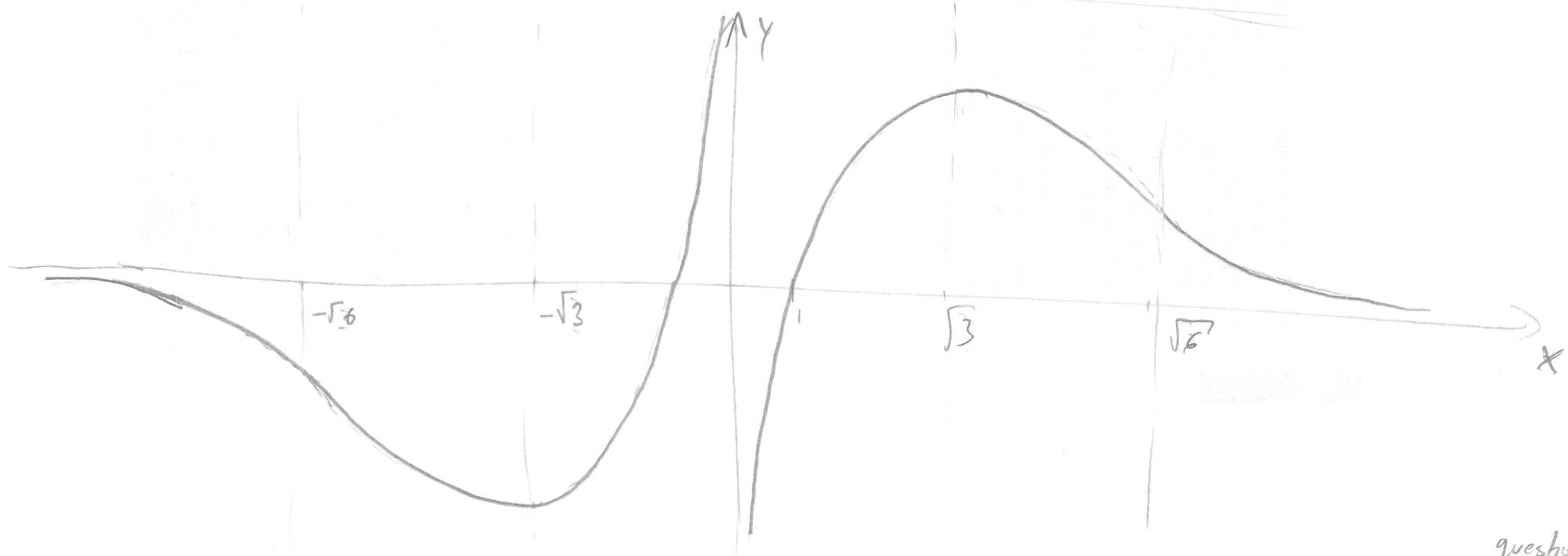


the sign of $f''(x)$ can change because x^5 can be positive or negative

$\lim\limits_{x\to\pm\infty} f(x) = 0 \Rightarrow$ H.A

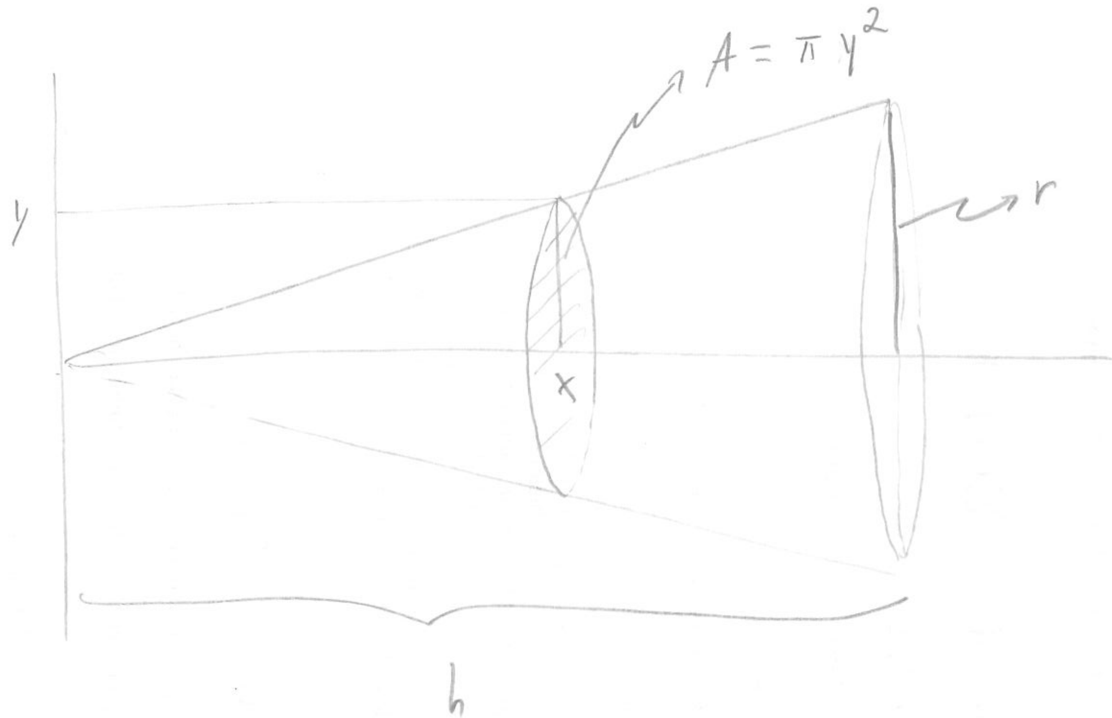
Question 9,1

		$-\sqrt{6}$	$-\sqrt{3}$	0	$\sqrt{3}$	$\sqrt{6}$
$f'(x)$	$-$	$+$	$+$	$+$	$-$	$-$
f''	$-$	$+$	$+$	$-$	$-$	$+$
shape						



question 9, 2

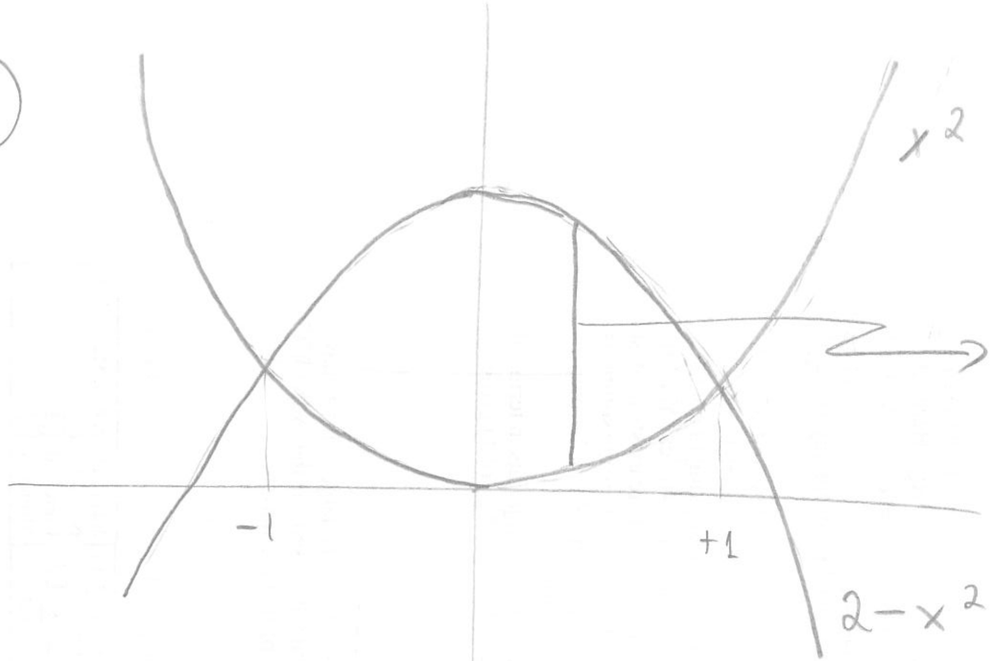
11



$$\frac{y}{x} = \frac{r}{h} \Longrightarrow y = \frac{rx}{h}, \quad A = \frac{\pi r^2}{h^2} x^2$$

$$V = \int_0^h A(x)\,dx = \frac{\pi r^2}{h^2} \int_0^h x^2\,dx = \frac{\pi r^2}{h^2} \frac{x^3}{3} \Big|_0^h = \frac{\pi r^2 h}{3}$$

12



length $= 2-x^2-x^2 = 2-2x^2$

$A = (2-2x^2)^2$

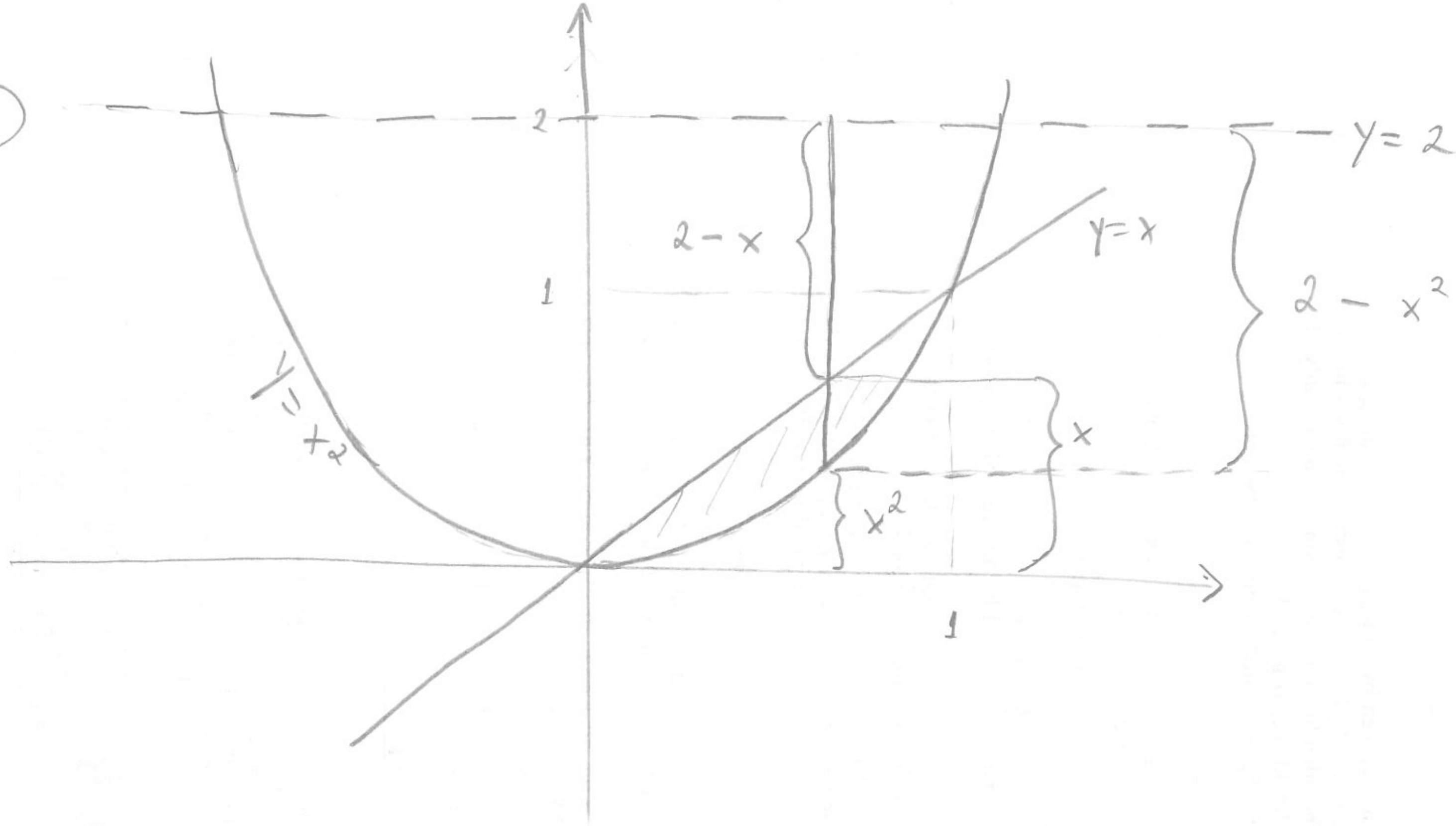
$x^2 = 2-x^2$

$\Rightarrow x^2 = 1, \; x = \pm 1$

$V = \int_{-1}^{1} (2-2x^2)^2 \, dx$

Question 12

13



A) about x: outer: $y = x$, inner $y = x^2$, $V = \int_0^1 \pi \left(x^2 - (x^2)^2 \right) dx$

B) about y: outer $y = x^2 \Rightarrow x = \sqrt{y}$, inner $y = x$, $V = \int_0^1 \pi \left((\sqrt{y})^2 - y^2 \right) dy$

C) about $y = 2$. outer: $2 - x^2$, inner: $2 - x$ } see picture

$$V = \int_0^1 \pi \left((2 - x^2)^2 - (2 - x)^2 \right) dx$$

question 13