MATH 155A FALL 13
PRACTICE MIDTERM 1.

Question 1. Find the domain of the following functions.

(a) $f(x) = \frac{2x^3-5}{x^2+x-6}$.

(b) $g(x) = \frac{x+1}{1+\frac{1}{x+1}}$.

(c) $f(x) = \sqrt{5-x} + \frac{1}{\sqrt{x-10}}$.

Question 2. An electricity company charges its customers a base rate of \$10 a month, plust 5 cents per kilowatt-hour (kWh) for the first 1200 kWh and 7 cents per kWh for all usage over 1200 kWh. Express the monthly cost E as a function of the amount x of electricity used.

Question 3. At the surface of the ocean, the water pressure is the same as the air pressure above the water, 15 lb/in^2. Below the surface, the water pressure increases by 4.34 lb/in^2 for every 10 ft of descent. Express the water pressure as a function of the depth below the ocean surface.

Question 4. Compute the values of the following trigonometric expressions.

(a) $\sin \frac{5\pi}{6}$.

(b) $\tan \frac{19\pi}{4} + \cos(-\frac{\pi}{6})$.

(c) $\sec \frac{4\pi}{3}$.

Question 5. Prove the following formulas.

(a) $\sin^2 x - \sin^2 y = \sin(x+y)\sin(x-y)$.

(b) $\cos^2 \theta = \frac{1+\cos(2\theta)}{2}$.

Question 6. Find all solutions to the following trigonometric equations.

(a) $2\cos x - 1 = 0$.

(b) $|\tan x| = 1$.

(c) $2 + \cos 2x = 3\cos x$.

Question 7. Evaluate the following limits, showing that the limit does not exist when that is the case.

(a) $\lim_{x\to 3^-} \dfrac{x+2}{x+3}$.

(b) $\lim_{x\to 1} \dfrac{x^3-1}{\sqrt{x}-1}$.

(c) $\lim_{x\to 2} \sqrt{\dfrac{2x^3+1}{3x-2}}$.

(d) $\lim_{x\to 0} \sqrt{x^3+x^2}\sin\dfrac{\pi}{x}$.

(e) $\lim_{x\to \frac{\pi}{2}} |\tan x|$.

(f) $\lim_{x\to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{|x|}\right)$.

(g) $\lim_{x\to 0} \left(\dfrac{1}{x} - \dfrac{1}{|x|}\right)$.

(h) $\lim_{x\to \pi} \sin(x+\sin x)$.

(i) $\lim_{x\to 7} \sqrt{1+\dfrac{1}{x}}$.

Question 8. Let

$$g(x) = \begin{cases} x & \text{if } x < 1, \\ 3 & \text{if } x = 1, \\ 2 - x^2 & \text{if } 1 < x \le 2, \\ x - 3 & \text{if } x > 2. \end{cases}$$

Evaluate or explain why the limit does not exist.

(a) $\lim_{x\to 1^-} g(x)$.

(b) $\lim_{x\to 1} g(x)$.

(c) $\lim_{x\to 2^-} g(x)$.

(d) $\lim_{x\to 2+} g(x)$.

(e) $\lim_{x\to 2} g(x)$.

Question 9. For the function g of the previous question, indicate the values of x for which g is not continuous.

Question 10. Explain why the following functions are continuous at every point in their domain.

(a) $f(x) = \dfrac{\sin x}{x+1}$.

(b) $f(x) = \dfrac{\tan x}{\sqrt{4-x^2}}$.

(c) $f(x) = \sin(\cos(\sin x))$.

Question 11. Let $f(x) = \frac{x^3-8}{x^2-4}$. Can you define a new function, $g(x)$, which agrees with $f(x)$ on the domain of $f(x)$ and is continuous at $x = 2$? What value should $f(2)$ have if we want to define it as a continuous function at $x = 2$?

Question 12. Using the ε, δ definition of a limit, show that

(a) $\lim_{x\to 10}(3 - \dfrac{4}{5}x) = -5$.

(b) $\lim_{x\to -6^+} \sqrt[8]{6+x} = 0$.

Question 13. Using ε, δ arguments, prove that the function $f(x) = \frac{1}{x+1}$ is continuous at every point on its domain.

Question 14. Using the definition of derivative, compute $f'(x)$.

(a) $f(x) = x^2$.

(b) $f(x) = \dfrac{1-2x}{3+x}$.

URL: http://www.disconzi.net/Teaching/MAT155A-Fall-13/MAT155A-Fall-13.html