MATH 155A FALL 13
EXAMPLES APPENDIX D

If $0 < \theta < \frac{\pi}{2}$ is such that

$$2(\cos\theta)^2 + (8 - \sqrt{3})\cos\theta - 4\sqrt{3} = 0,$$

find θ.

Solution. Rewrite the equation as

$$(\cos\theta)^2 + \frac{8 - \sqrt{3}}{2}\cos\theta - 2\sqrt{3} = 0. \tag{1}$$

Using the quadratic formula, we see that the equation

$$x^2 + \frac{8 - \sqrt{3}}{2}x - 2\sqrt{3} = 0$$

has solutions

$$x = \frac{\sqrt{3}}{2} \qquad \text{and} \qquad x = -4.$$

Hence (1) can be written as

$$\Big(\cos\theta - \frac{\sqrt{3}}{2}\Big)\Big(\cos\theta + 4\Big) = 0,$$

which implies $\cos\theta = \frac{\sqrt{3}}{2}$ or $\cos\theta = -4$. This last condition is never satisfied since $-1 \leq \cos\theta \leq 1$ for any theta, so we conclue that

$$\cos\theta = \frac{\sqrt{3}}{2}.$$

The only θ between zero and $\frac{\pi}{2}$ satisfying the above is $\frac{\pi}{6}$, thus we conclude

$$\theta = \frac{\pi}{6}.$$

URL: `http://www.disconzi.net/Teaching/MAT155A-Fall-13/MAT155A-Fall-13.html`