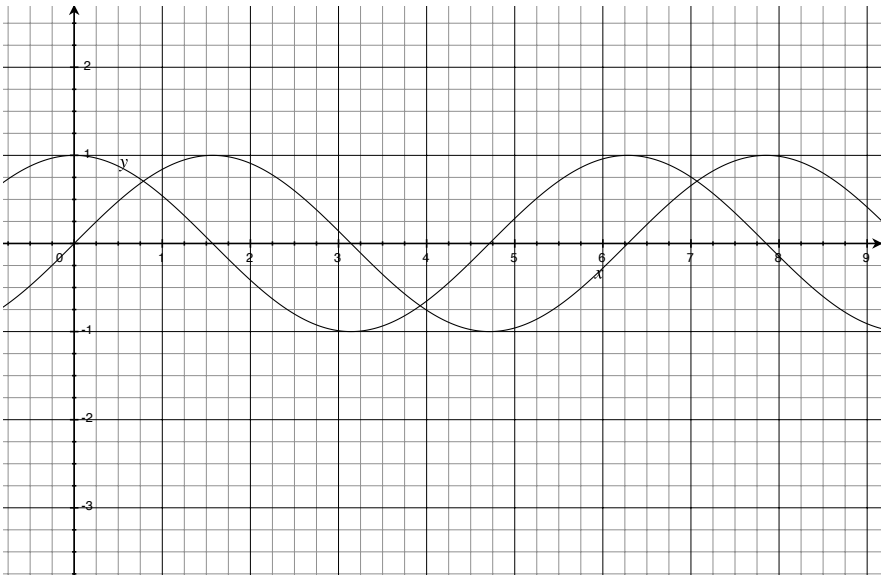
MATH 155A FALL 13
EXAMPLES SECTION 5.1.

Question. Find the area between the curves $y = \sin x$ and $y = \cos x$, where $0 \leq x \leq 2\pi$.

Solution. We start graphing the functions:



We see that $\cos x \geq \sin x$ for $0 \leq x \leq \frac{\pi}{4}$ and for $\frac{5\pi}{4} \leq x \leq 2\pi$, and $\sin x \geq \cos x$ for $\frac{\pi}{4} \leq x \leq \frac{5\pi}{4}$. Therefore

$$\begin{aligned}
\int_0^{2\pi} |\cos x - \sin x| dx &= \int_0^{\frac{\pi}{4}} |\cos x - \sin x| dx + \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} |\cos x - \sin x| dx + \int_{\frac{5\pi}{4}}^{2\pi} |\cos x - \sin x| dx \\
&= \int_0^{\frac{\pi}{4}} (\cos x - \sin x) dx + \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x) | dx + \int_{\frac{5\pi}{4}}^{2\pi} (\cos x - \sin x) dx \\
&= \sin x \Big|_0^{\frac{\pi}{4}} - (-\cos x) \Big|_0^{\frac{\pi}{4}} + (-\cos x) \Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}} - \sin x \Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}} + \sin x \Big|_{\frac{5\pi}{4}}^{2\pi} - (-\cos x) \Big|_{\frac{5\pi}{4}}^{2\pi} \\
&= (\frac{\sqrt{2}}{2} - 0) - (-\frac{\sqrt{2}}{2} + 1) - (\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}) - (-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}) \\
&+ (0 + \frac{\sqrt{2}}{2}) - (-1 + \frac{\sqrt{2}}{2}) \\
&= \sqrt{2}.
\end{aligned}$$

URL: `http://www.disconzi.net/Teaching/MAT155A-Fall-13/MAT155A-Fall-13.html`