MATH 155A FALL 13
EXAMPLES SECTION 3.9.

Question. Find the particular anti-derivative of the following functions:
(a) $\frac{1}{\tan x} \cos^{-3} x \, \sin x + 1$.
(b) $2x \cos x - x^2 \sin x$.

Solutions.

(a) Simplifying:

$$\begin{aligned} \frac{1}{\tan x} \cos^{-3} x \, \sin x &= \frac{\cos x}{\sin x} \frac{\sin x}{\cos^3 x} \\ &= \frac{1}{\cos^2 x} \\ &= \sec^2 x, \end{aligned}$$

whose particular anti-derivative is $\tan x$. The particular anti-derivative of 1 is x since $x' = 1$ (this can also be obtained from the rule for x^n with $n = 0$).

Hence

$$\tan x + x$$

is the particular anti-derivative we are looking for.

(b) Put $f(x) = 2x \cos x - x^2 \sin x$. This does not look like any of the functions whose anti-derivatives were given in class. So we start noticing that it involves the product of functions, hence we suspect that the particular anti-derivative of f, denoted by F, should also involve the product of functions. Write $F = F_1 F_2$. By the product rule,

$$F' = F_1' F_2 + F_1 F_2'$$

and we want this to be equal to f. Hence we try to match $F_1' F_2 = 2x \cos x$ and $F_1 F_2' = -x^2 \sin x$. Setting $F_1 = x^2$ and $F_2 = \cos x$, we see that

$$(x^2 \cos x)' = 2x \cos x - x^2 \sin x,$$

and thus $F(x) = x^2 \cos x$.

Remark: we shall soon learn techniques to find the anti-derivatives of more complicated functions that do not involve some "guessing" as in the previous example.

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