MATH 155A FALL 13
EXAMPLES SECTION 2.9.

Question. Use linear approximation to estimate $\sqrt{99.8}$.

Solution.

Put $f(x) = \sqrt{x}$, so that $f'(x) = \frac{1}{2\sqrt{x}}$. We want $\sqrt{99.8} = \sqrt{100 - 0.2}$, hence we take $a = 100$:

$$f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{20},$$

and then

$$\begin{aligned} L(x) &= f(a) + f'(a)(x - a) = \sqrt{100} + \frac{1}{20}(x - 100) \\ &= 10 + \frac{1}{20}(x - 100). \end{aligned}$$

But $f(x) \approx L(x)$ for x near $a = 100$, thus

$$\begin{aligned} f(99.8) = \sqrt{99.8} \approx L(99.8) &= 100 + \frac{1}{20}(99.8 - 100) \\ &= 10 - 0.01 = 9.99. \end{aligned}$$

Another method: we could have used differentials. Put $y = \sqrt{x}$, so $dy = \frac{1}{2\sqrt{x}}dx$. When $x = 100$ and $dx = -0.2$ we get $dy = \frac{1}{2\sqrt{100}}(-0.2) = -0.01$. hence

$$\sqrt{99.8} = y(99.8) \approx y(100) + dy = 10 - 0.01 = 9.99.$$

URL: `http://www.disconzi.net/Teaching/MAT155A-Fall-13/MAT155A-Fall-13.html`