MATH 155A FALL 13
EXAMPLES SECTIONS 2.6.

Question. If $x^2 + xy + y^3 = 1$, find y'' at $x = 1$.

Solution. Differentiating both sides of the equation produces

$$2x + y + xy' + 3y^2y' = 0. \tag{1}$$

Differentiating again

$$2 + 2y' + xy'' + 6y(y')^2 + 3y^2y'' = 0.$$

Hence,

$$y'' = -\frac{6y(y')^2 + 2y' + 2}{3y^2 + x}. \tag{2}$$

Since we want to plug in $x = 1$ to find $y''(1)$, we need the values of y and y' when $x = 1$, as these terms appear on the right hand side of the above expression for y''.

Plugging $x = 1$ in the original equation gives

$$1 + y^3 + y = 1 \Rightarrow y(1) = 0.$$

From equation (1) we find

$$y' = -\frac{2x + y}{x + 3y^2}.$$

Plugging $x = 1$ and $y = 0$ tells us that $y'(1) = -2$. Plugging $x = 1$, $y = 0$, $y' = -2$ into (2) finally gives $y''(1) = 2$.

URL: `http://www.disconzi.net/Teaching/MAT155A-Fall-13/MAT155A-Fall-13.html`