MATH 155A FALL 13
EXAMPLES SECTIONS 2.4 AND 2.5.

1. For each function below, write it as composition of elementary functions whose derivatives can be computed by direct applications of the formulas seen in class. Then compute their derivative.

(a) $\sec x$.

(b) $\sqrt{x^5 - x^2}$.

(c) $\frac{1}{(\sin x + 2x)^3}$.

(d) $\frac{1}{\tan \sqrt{x^2+1}}$.

Solutions.

In all questions we shall employ the chain rule:

$$\big(f(g(x))\big)' = f'(g(x))\, g'(x).$$

(a) Recall that $\sec x = \frac{1}{\cos x}$, so $\sec x = f(g(x))$ with $f(x) = \frac{1}{x}$ and $g(x) = \cos x$. Now

$$f'(x) = -\frac{1}{x^2},$$
$$g'(x) = -\sin x,$$

thus

$$(\sec x)' = -\frac{1}{\cos^2 x}(-\sin x) = \frac{\sin x}{\cos^2 x}.$$

This can be written as

$$(\sec x)' = \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x}\frac{\sin x}{\cos x} = \sec x \tan x.$$

(b) $\sqrt{x^5 - x^2} = f(g(x))$ with $f(x) = \sqrt{x}$ and $g(x) = x^5 - x^2$. Now

$$f'(x) = \frac{1}{2\sqrt{x}},$$
$$g'(x) = 5x^4 - 2x,$$

thus

$$(\sqrt{x^5 - x^2})' = \frac{1}{2\sqrt{x^5 - x^2}}(5x^4 - 2x) = \frac{5x^4 - 2x}{2\sqrt{x^5 - x^2}}.$$

(c) Write

$$f(x) = \frac{1}{(\sin x + 2x)^3} = (\sin x + 2x)^{-3} = f(g(x))$$

with $f(x) = x^{-3}$ and $g(x) = \sin x + 2x$. Then

$$f'(x) = -3x^{-4},$$
$$g'(x) = \cos x + 2.$$

So,

$$\left(\frac{1}{(\sin x + 2x)^3}\right)' = -\frac{3\cos x + 6}{(\sin x + 2x)^4}.$$

(d) Notice that we have the composition of more than two functions here. In fact

$$\frac{1}{\tan\sqrt{x^2+1}} = f(g(h(u(x)))),$$

where

$$f(x) = \frac{1}{x},$$
$$g(x) = \tan x,$$
$$h(x) = \sqrt{x},$$
$$u(x) = x^2 + 1.$$

A successive applications of the chain rule yields

$$\begin{aligned}\big(f(g(h(u(x))))\big)' &= f'(g(h(u(x))))\big(g(h(u(x)))\big)' \\ &= f'(g(h(u(x))))g'(h(u(x)))\big(h(u(x))\big)' \\ &= f'(g(h(u(x))))g'(h(u(x)))h'(u(x))u'(x).\end{aligned}$$

Computing

$$f'(x) = -\frac{1}{x^2},$$
$$g'(x) = \sec^2 x,$$
$$h'(x) = \frac{1}{2\sqrt{x}},$$
$$u'(x) = 2x.$$

Hence

$$\begin{aligned}\left(\frac{1}{\tan\sqrt{x^2+1}}\right)' &= -\frac{1}{\tan^2\sqrt{x^2+1}}\sec^2\sqrt{x^2+1}\,\frac{1}{2\sqrt{x^2+1}}2x \\ &= -\frac{x\sec^2\sqrt{x^2+1}}{\sqrt{x^2+1}\tan^2\sqrt{x^2+1}}.\end{aligned}$$

URL: `http://www.disconzi.net/Teaching/MAT155A-Fall-13/MAT155A-Fall-13.html`