

MATH 155A FALL 13
EXAMPLES SECTIONS 2.1 AND 2.2.

Question. Using the definition of derivative, compute $f'(x)$ if $f(x) = \frac{x^2+2}{x+3}$.

Solution. Write

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \rightarrow 0} \frac{\frac{(x+h)^2+2}{(x+h)+3} - \frac{x^2+2}{x+3}}{h} \\ &= \lim_{h \rightarrow 0} \left[\frac{1}{h} \left(\frac{(x+h)^2+2}{x+h+3} - \frac{x^2+2}{x+3} \right) \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{1}{h} \frac{((x+h)^2+2)(x+3) - (x^2+2)(x+h+3)}{(x+h+3)(x+3)} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{1}{h} \frac{(x^2+2xh+h^2+2)(x+3) - (x^2+2)(x+h+3)}{(x+h+3)(x+3)} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{1}{h} \frac{(x^2+2)(x+3) + (2xh+h^2)(x+3) - (x^2+2)(x+3) - (x^2+2)h}{(x+h+3)(x+3)} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{1}{h} \frac{(2xh+h^2)(x+3) - (x^2+2)h}{(x+h+3)(x+3)} \right] \\ &= \lim_{h \rightarrow 0} \frac{(2x+h)(x+3) - (x^2+2)}{(x+h+3)(x+3)} \\ &= \frac{2x(x+3) - (x^2+2)}{(x+3)^2} \\ &= \frac{x^2+6x-2}{(x+3)^2}.\end{aligned}$$

URL: <http://www.disconzi.net/Teaching/MAT155A-Fall-13/MAT155A-Fall-13.html>