MATH 155A FALL 13
EXAMPLES OF SECTIONS 1.5 AND 1.6

Evaluate the following limits, when possible.

(a)

$$\lim_{x\to 2} 3x$$

(b)

$$\lim_{x\to 0} \frac{x}{|x|}.$$

(c)

$$\lim_{x\to 3} \frac{x^2-2x-3}{x^2-4x+3}.$$

Solutions.

(a) When x approaches 2, $3x$ approaches 6, with no undefined expressions arising. Hence

$$\lim_{x\to 2} 3x = 6.$$

(b) Recall that

$$|x| = \begin{cases} x, & x \geq 0, \\ -x, & x < 0. \end{cases}$$

Hence, if we approach zero through positive values

$$\begin{aligned} \lim_{x\to 0+} \frac{x}{|x|} &= \lim_{x\to 0+} \frac{x}{x} \\ &= \lim_{x\to 0+} 1 = 1. \end{aligned}$$

On the other hand, approaching from negative values

$$\begin{aligned} \lim_{x\to 0-} \frac{x}{|x|} &= \lim_{x\to 0-} \frac{x}{-x} \\ &= \lim_{x\to 0-} -1 = -1. \end{aligned}$$

Therefore the limits from the left and right sides do not agree, and $\lim_{x\to 0} \frac{x}{|x|}$ does not exit.

(c) Notice that the denominator x^2-4x+3 approaches zero when $x \to 3$, so we cannot plug in $x = 3$. But

$$\lim_{x\to 3} \frac{x^2-2x-3}{x^2-4x+3} = \lim_{x\to 3} \frac{(x-3)(x+1)}{(x-3)(x-1)} = \lim_{x\to 3} \frac{x+1}{x-1} = \frac{3+1}{3-1} = 2.$$