

MATH 155A FALL 13
EXAMPLES OF SECTIONS 1.1 AND 1.2

Find the domain of the following functions.

(a) $f(x) = x^3 + 2x - 7$.

(b) $g(x) = \frac{3x}{\sqrt{x^3 - 4x}}$.

Solutions.

(a) This function is a polynomial, hence its domain is \mathbb{R} . Alternatively, notice that given any real number x , its cubic power x^3 and multiplication by 2, i.e., $2x$, are well defined.

(b) Since we cannot take the square root of a negative number, and the denominator of a fraction cannot be equal to zero, we conclude that x must satisfy

$$x^3 - 4x > 0.$$

Write this as

$$x(x^2 - 4) > 0. \tag{1}$$

For a product ab to be positive we need both a and b to be positive, or both a and b to be negative. Hence (1) holds if and only if

$$x^2 - 4 > 0 \quad \text{and} \quad x > 0, \tag{2}$$

or

$$x^2 - 4 < 0 \quad \text{and} \quad x < 0. \tag{3}$$

Looking at the first inequality in (2), we have

$$x^2 > 4 \Rightarrow x > 2 \text{ or } x < -2.$$

Since in (2) both $x^2 - 4 > 0$ and $x > 0$ have to hold simultaneously, we conclude that $x > 2$ solves (2).

Looking now at the first inequality in (3), we have

$$x^2 < 4 \Rightarrow -2 < x < 2.$$

Since in (3) both $x^2 - 4 < 0$ and $x < 0$ have to hold simultaneously, we conclude that $-2 < x < 0$ solves (3).

Therefore the domain of g is given by

$$(-2, 0) \cup (2, \infty).$$