Stony Brook University.

MAT 127 — Calculus C, Spring 12.

Review sheet for the first midterm — series

This sheet is a quick summary of important results for series. By no means it should be seen as a replacement for studying the textbook and class notes. Notice that sequences and remainder estimates will also be in the exam, even though they are not presented here. All references to pages and examples are from the textbook.

Important Series.

Geometric series:

$$\sum_{n=0}^{\infty} ar^n = \begin{cases} \frac{a}{1-r} & \text{if } |r| < 1, \\ \text{divergent} & \text{if } |r| \geq 1. \end{cases}$$

See example 2 on p. 567, and example 3 on p. 568.

Harmonic series:

$$\sum_{n=1}^{\infty} \frac{1}{n} = \infty \text{ (divergent)}$$

p-series:

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \begin{cases} \text{convergent} & \text{if } p > 1, \\ \text{divergent} & \text{if } p \leq 1. \end{cases}$$

Alternating series: a series where the terms alternate between positive and negative,

$$\sum_{n=1}^{\infty} (-1)^n b_n \text{ where } b_n > 0.$$

Notice that $(-1)^n = 1$ if n is even and $(-1)^n = -1$ if n is odd.

Telescoping series: a series where the n^{th} term cancels with the $(n+1)^{th}$ term, e.g.

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right) = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{3} + \frac{1}{4} - \dots$$

See example 6, p. 568.

Convergence tests for series.

Divergence test: if $\lim_{n\to\infty} a_n$ does not exist or $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ diverges. VERY COMMON MISTAKE: if $\lim_{n\to\infty} a_n = 0$, you *cannot* conclude that the series converges. For example, $\lim_{n\to\infty} \frac{1}{n} = 0$ but the harmonic series diverges.

Integral test: $f(x)$ a function which is continuous, positive and decreasing for $x \geq 1$. $a_n = f(n)$. Then

$$(a) \text{ if } \int_1^{\infty} f(x)dx \text{ converges, then } \sum_{n=1}^{\infty} a_n \text{ converges.}$$
$$(b) \text{ if } \int_1^{\infty} f(x)dx \text{ diverges, then } \sum_{n=1}^{\infty} a_n \text{ diverges.}$$

See example 1 p. 577.

Comparison test: a_n and b_n are *positive* terms.

$$(a) \text{ if } \sum_{n=1}^{\infty} b_n \text{ converges and } a_n \leq b_n \text{ then } \sum_{n=1}^{\infty} a_n \text{ converges.}$$
$$(b) \text{ if } \sum_{n=1}^{\infty} b_n \text{ diverges and } a_n \geq b_n \text{ then } \sum_{n=1}^{\infty} a_n \text{ diverges.}$$

See examples 2 and 4 on p. 579. COMMON MISTAKE: if $\sum_{n=1}^{\infty} b_n$ diverges and $a_n \leq b_n$ then you *cannot* conclude that $\sum_{n=1}^{\infty} a_n$ diverges; if $\sum_{n=1}^{\infty} b_n$ converges and $a_n \geq b_n$ then you *cannot* conclude that $\sum_{n=1}^{\infty} a_n$ converges.

Limit comparison test: a_n and b_n positive terms. If

$$\lim_{n \to \infty} \frac{a_n}{b_n} = c$$

where c is not ∞ neither zero, then either both series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge or both diverge. See example 5 on p. 580.

Ratio test:

$$(a) \text{ if } \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1 \text{ then } \sum_{n=1}^{\infty} a_n \text{ is absolutely convergent and convergent.}$$
$$(b) \text{ if } \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1 \text{ then } \sum_{n=1}^{\infty} a_n \text{ is divergent.}$$
$$(c) \text{ if } \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1 \text{ then the test doesn't give any information.}$$

See example 8, p. 590. COMMON MISTAKE: it must be $\frac{a_{n+1}}{a_n}$ and *not* $\frac{a_n}{a_{n+1}}$.

Alternating series test: $\sum_{n=1}^{\infty} (-1)^n b_n$, where $b_n > 0$. If

$$(a) \; b_{n+1} \leq b_n \text{ for all } n, \text{ and}$$
$$(b) \lim_{n \to \infty} b_n = 0,$$

then the series converges. See examples 1, 2 and 3 on p. 586.