Stony Brook University.
MAT 127 — Calculus C, Spring 12.
Examples for section 7.2

PROBLEMS

Question 1. Use Euler's method with step size 0.5 to compute the approximate y-values y_1, y_2, y_3 and y_4 for the solution of the initial value problem $y' = y - 2x$, $y(1) = 0$.

Question 2. Use Euler's method with step size 0.2 to estimate $y(1)$, where $y(x)$ is the solution of the initial value problem $y' + 3x^2y = 6x^2$, $y(0) = 3$.

For the questions above, recall that if

$$\begin{cases} y' = F(x, y) \\ y(x_0) = y_0 \end{cases}$$

then the formula for the Euler's method with step size h is

$$\begin{cases} x_{n+1} = x_n + h \\ y_{n+1} = y_n + hF(x_n, y_n) \end{cases}$$

SOLUTIONS

1. We have $F(x, y) = y - 2x$, $x_0 = 1$, $y_0 = 0$. Then

$$\begin{cases} x_1 = 1 + 0.5 = 1.5 \\ y_1 = 0 + 0.5(0 - 2 \cdot 1) = -1 \end{cases}$$

$$\begin{cases} x_2 = 1.5 + 0.5 = 2 \\ y_2 = -1 + 0.5(-1 - 2 \cdot 1.5) = -3 \end{cases}$$

$$\begin{cases} x_3 = 2.0 + 0.5 = 2.5 \\ y_3 = -3 + 0.5(-3 - 2 \cdot 2) = -6.5 \end{cases}$$

$$\begin{cases} x_4 = 2.5 + 0.5 = 3 \\ y_4 = -6.5 + 0.5(-6.5 - 2 \cdot 2.5) = -12.25 \end{cases}$$

2. We have $F(x,y) = 6x^2 - 3x^2y$, $x_0 = 0$, $y_0 = 3$. Then

$$\begin{cases} x_1 = 0 + 0.2 = 0.2 \\ y_1 = 3 + 0.2(6 \cdot 0^2 - 3 \cdot 0^2 \cdot 3) = 3 \end{cases}$$

$$\begin{cases} x_2 = 0.2 + 0.2 = 0.4 \\ y_2 = 3 + 0.2(6 \cdot (0.2)^2 - 3 \cdot (0.2)^2 \cdot 3) = 2.98 \end{cases}$$

$$\begin{cases} x_3 = 0.4 + 0.2 = 0.6 \\ y_3 = 2.98 + 0.2(6 \cdot (0.4)^2 - 3 \cdot (0.4)^2 \cdot 2.98) = 2.89 \end{cases}$$

$$\begin{cases} x_4 = 0.6 + 0.2 = 0.8 \\ y_4 = 2.89 + 0.2(6 \cdot (0.6)^2 - 3 \cdot (0.6)^2 \cdot 2.89) = 2.70 \end{cases}$$

$$\begin{cases} x_5 = 0.8 + 0.2 = 1.0 \\ y_5 = 2.70 + 0.2(6 \cdot (0.8)^2 - 3 \cdot (0.8)^2 \cdot 2.70) = 2.43 \end{cases}$$

Hence $y(1) \approx 2.43$.