MAT123 - Introduction to calculus

Some examples and hints for the First Midterm

The First Midterm will be on Thursday Feb 25th, 8:25pm, at Javis 100.

IMPORTANT: More examples can be found under "Announcements and extra material" at `http://www.math.sunysb.edu/~disconzi/Teaching/MAT123-Spring-10/MAT123-Spring-10.html`

You should do ALL the following review problems from the textbook:
CHAPTER 4 REVIEW: 32, 33, 34, 60, 61, 63, 66, 68, 77, 83, 97, 104
CHAPTER 5 REVIEW: 51, 53, 55, 59

*If you are using the old (second) edition, do these problems instead:
Page 564-566: 32, 33, 34, 60, 61, 63, 66, 68, 83, 93, 97, 104
Page 548-549: 99, 100, 101, 102, 103, 104, 105, 106

Question 1. Compute:
(a) $\sin 330°$
Solution: The reference angle to $330°$ is $30°$. The angle $330°$ is on the fourth quadrant and on the fourth quadrant sin is negative. Therefore $\sin 330° = -\sin 30° = -\frac{1}{2}$.
(b) $\cos\frac{9\pi}{4}$
Solution: $\frac{9\pi}{4} = 2\pi + \frac{\pi}{4}$. By periodicity we can disregard the factor 2π, hence $\cos\frac{9\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.
(c) $\tan^{-1}(-\sqrt{3})$
Solution: From our table of values we have $\tan^{-1}(\sqrt{3}) = 60°$. $\tan^{-1}$ has range $(-90°, 90°)$ and tan is negative on $(-90°, 0)$, so $\tan^{-1}(-\sqrt{3}) = -60°$.
(d) $\sec\frac{2\pi}{3}$
Solution: $\sec x = \frac{1}{\cos x}$. The reference angle for $\frac{2\pi}{3}$ is $\frac{\pi}{3}$ (because $\frac{2\pi}{3} + \frac{\pi}{3} = \pi$). $\frac{2\pi}{3}$ is on the second quadrant and on the second quadrant cos is negative, so $\cos\frac{2\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$. Therefore $\sec\frac{2\pi}{3} = -2$.
(e) $\csc^{-1}(2)$
Solution: $\csc^{-1}(2)$ means: the value of the angle θ such that $\csc\theta = 2$. Because $\csc\theta = \frac{1}{\sin\theta}$, we have that $\csc\theta = 2$ is the same as $\sin\theta = \frac{1}{2}$, which gives $\theta = 30°$.

Question 2. Solve for x the equation:

$$\cos^2 x = \frac{1}{2}$$

Solution: Taking the square root:

$$\sqrt{\cos^2 x} = \sqrt{\frac{1}{2}} \Rightarrow \cos x = \pm\frac{\sqrt{2}}{2}$$

Separate the plus and minus cases. Each of them gives *two* answers between 0 and 2π.
CASE 1: $\cos x = +\frac{\sqrt{2}}{2}$. The values of x (between 0 and 2π) which give $\cos x = \frac{\sqrt{2}}{2}$ are $x = 45° = \frac{\pi}{4}$ and $x = 315° = \frac{7\pi}{4}$.
CASE 2: $\cos x = -\frac{\sqrt{2}}{2}$. The values of x (between 0 and 2π) which give $\cos x = -\frac{\sqrt{2}}{2}$ are

$x = 135° = \frac{3\pi}{4}$ and $x = 225° = \frac{5\pi}{4}$.
Now we have to add full rotations to those values, so the answer is:
$x = \frac{\pi}{4} + 2\pi k,\ k = \pm 0, 1, 2 \ldots$
$x = \frac{3\pi}{4} + 2\pi k,\ k = \pm 0, 1, 2 \ldots$
$x = \frac{5\pi}{4} + 2\pi k,\ k = \pm 0, 1, 2 \ldots$
$x = \frac{7\pi}{4} + 2\pi k,\ k = \pm 0, 1, 2 \ldots$

Question 3. Graph three cycles of $y = \sin(2x + \frac{\pi}{2})$
Solution:

STEP 0: Identify A, B and C. Write $y = A\sin(Bx - C)$, and compare with with $y = \sin(2x + \frac{\pi}{2})$ to find A, B and C. We obtain $A = 1$, $B = 2$ and $C = -\frac{\pi}{2}$ (notice that because the formula we studied in class has a *minus* in front of C and in this problem we have a *plus* in front of $\frac{\pi}{2}$ we conclude that C is negative, i.e., $C = -\frac{\pi}{2}$).

STEP 1: Find the period and the amplitude. The amplitude is given by the formula Amplitude=$|A|$, so in our case the amplitude is 1. This means that the graph will be between -1 and 1 (if the amplitude were 2 the graph would be between -2 and 2). The period is given by the formula $P = \frac{2\pi}{B}$, in our case $P = \frac{2\pi}{B} = \frac{2\pi}{C} = \pi$.

STEP 2: Find the values of x for the five key points. We have to use the formulas:

$$x_1 = \frac{C}{B}, \quad x_2 = x_1 + \frac{\text{period}}{4}, \quad x_3 = x_2 + \frac{\text{period}}{4}, \quad x_4 = x_3 + \frac{\text{period}}{4}, \quad x_5 = x_4 + \frac{\text{period}}{4}$$

So in our case we have:

$$x_1 = \frac{C}{B} = -\frac{\pi}{4}$$
$$x_2 = x_1 + \frac{\text{period}}{4} = -\frac{\pi}{4} + \frac{\pi}{4} = 0$$
$$x_3 = x_2 + \frac{\text{period}}{4} = 0 + \frac{\pi}{4} = \frac{\pi}{4}$$
$$x_4 = x_3 + \frac{\text{period}}{4} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$
$$x_5 = x_4 + \frac{\text{period}}{4} = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$

STEP 3: Find the values of y for the five key points. Here we have to find the y value corresponding to each one of the points in STEP 2.

$$x_1 = -\frac{\pi}{4} \Rightarrow y_1 = \sin(2x_1 + \frac{\pi}{2}) = \sin(2 \times (-\frac{\pi}{4}) + \frac{\pi}{2}) = \sin 0 = 0$$
$$x_2 = 0 \Rightarrow y_2 = \sin(2x_2 + \frac{\pi}{2}) = \sin(2 \times 0 + \frac{\pi}{2}) = \sin\frac{\pi}{2} = 1$$
$$x_3 = \frac{\pi}{4} \Rightarrow y_3 = \sin(2x_3 + \frac{\pi}{2}) = \sin(2 \times \frac{\pi}{4} + \frac{\pi}{2}) = \sin(\pi) = 0$$
$$x_4 = \frac{\pi}{2} \Rightarrow y_4 = \sin(2x_4 + \frac{\pi}{2}) = \sin(2 \times \frac{\pi}{2} + \frac{\pi}{2}) = \sin\frac{3\pi}{2} = -1$$
$$x_5 = \frac{3\pi}{4} \Rightarrow y_5 = \sin(2x_5 + \frac{\pi}{2}) = \sin(2 \times \frac{3\pi}{4} + \frac{\pi}{2}) = \sin 2\pi = 0$$

As other examples, if we had $A = 2$ we would obtain $y_2 = 2$ and $y_4 = -2$, and if we had $A = -3$ we would obtain $y_2 = -3$ and $y_4 = 3$.

STEP 4: PLOT THE POINTS (x, y) AND GRAPH A CYCLE. Now plot the points $(x_1, y_1), (x_2, y_2), \ldots, (x_5, y_5)$ as in figure 1(a) and join them by a smooth curve like sine, as in figure 1(b).

STEP 5: REPEAT THE GRAPH AS MANY CYCLES AS REQUIRED. Repeat the graph twice to get three cylces, as in figure 1(c) (the graphs are at the end).

Some hints and remarks:

- Make sure that you know how to use reference angles to obtain values of any trig function from the basic values $30°, 45°, 60°$.
- Understand how to get all values of x in equations such as the one in question 2 above.

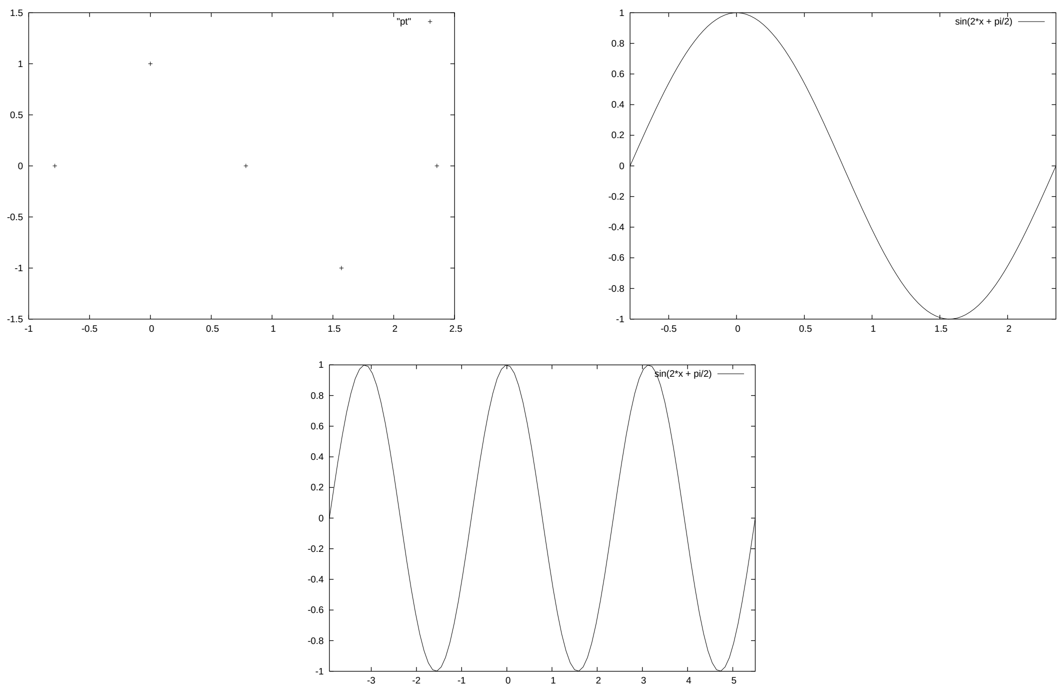

Figure 1: Steps in drawing the graph.