HOMEWORK 10

MATH 3120

Unless stated otherwise, the notation and terminology below is the same used in class.

Problem 1. In the equations below, identify the functions $a(t,x,u)$, $b(t,x,u)$, and $c(t,x,u)$ and write the corresponding characteristic system.

(a) $(1+t^2)\partial_t u + 3\partial_x u + u^2 = 0$.

(b) $\sin(x)e^t u_t + |u|^3 u_x = 0$.

Problem 2. Solve the problem below using the method of characteristics and give a description of the (projected) characteristics.

$$\begin{aligned} x\partial_t u - t\partial_x u - u &= 0, \\ u(0,x) &= h(x). \end{aligned}$$

Problem 3. Does the transversality condition hold for the problem of question 2? What can you say about uniqueness and how is it related to the solution you found?

Problem 4. Solve the following problems. In each case, sketch the characteristic curves, and indicate the region in the xy-plane where the solution is defined.

(a) $u_x + u_y = u^2$,

for (x,y) in the region $\{y \geq 0\}$, with the condition $u(x,0) = g(x)$, where g is a given function. Find the solution in the case $g(x) = x^2$.

(b) $u_x + u_y + u = 1$,

with the condition $u = \sin x$ on $y = x + x^2$, $x > 0$.

Problem 5. Solve

$$uu_x - uu_y = u^2 + (x+y)^2,$$

with initial condition $u(x,0) = 1$.

Hint: after writing the characteristic equations, identify an equation satisfied by $x + y$.