# HOMEWORK 1

MATH 3120

Unless stated otherwise, the notation and terminology below is the same used in class.

**Problem 1.** Review multivariable calculus, especially the chain rule in several variables and vector identities/operators.

**Problem 2.** Verify whether the given function is a solution of the given PDE:

(a) $u(x, y) = y \cos x + \sin y \sin x$, $u_{xx} + u = 0$.

(b) $u(x, y) = \cos x \sin y$, $(u_{xx})^2 + (u_{yy})^2 = 0$.

**Problem 3.** Determine whether the PDEs below are linear or nonlinear:

(a) $\dfrac{\partial^2 u}{\partial t^2} + e^t \dfrac{\partial u}{\partial x} + u = 0$.

(b) $\partial_x u \partial_y u = 1$.

(c) $\dfrac{\partial^2 z}{\partial t^2} + e^t \dfrac{\partial z}{\partial x} + \cos z = 0$.

(d) $(u_{xx})^2 + (u_{yy})^2 = 1$.

**Problem 4.** Consider Maxwell's equations:

$$\begin{aligned} \operatorname{div} E &= \frac{\varrho}{\varepsilon_0}, \\ \operatorname{div} B &= 0, \\ \frac{\partial B}{\partial t} + \operatorname{curl} E &= 0, \\ \frac{\partial E}{\partial t} - \frac{1}{\mu_0 \varepsilon_0} \operatorname{curl} B &= -\frac{1}{\varepsilon_0} J. \end{aligned}$$

Assume that $\varrho$ and $J$ vanish. Show that Maxwell's equations then imply that $E$ and $B$ satisfy the wave equation:

$$\frac{\partial^2 E}{\partial t^2} - \frac{1}{\varepsilon_0 \mu_0} \Delta E = 0,$$

and

$$\frac{\partial^2 B}{\partial t^2} - \frac{1}{\varepsilon_0 \mu_0} \Delta B = 0.$$

Interpret your result. Can you guess what the constant $\frac{1}{\varepsilon_0 \mu_0}$ must equal to?